Introduction to Statistics and Data Science using *eStat*

## Chapter 5 Probability Distribution

# 5.2 Calculation of Probability

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# 5.2 Calculation of Probability

## ❖ Permutation

- The number of ways to select r objects out of n objects considering the order is called permutation.

$$ {}_nP_r = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!} $$

- The number of ways to list all n objects is

$$ {}_nP_n = n(n-1)(n-2)\cdots 2\cdot 1 = n! $$

$0\,! = 1$

# 5.2 Calculation of Probability

## ❖ Combination

- **The number of ways to select r objects out of n objects without considering the order is called combination.**

$$ {}_{n}C_{r} = \frac{{}_{n}P_{r}}{r!} = \frac{n!}{r!(n-r)!} $$

# 5.2 Calculation of Probability

[Example 5.2.1] Four people A, B, C and D are placed on four chairs.

- Obtain the total number of cases in which four people are placed, and the number in which A is placed on the leftmost.
- What is the probability that A is placed on the leftmost side?

<Answer>

- The number of elements in the sample space is:
  (Number of people that can be placed on the leftmost)
  × (number of people except left who can be placed in the second position)
  × (number of people who can be placed in third place except for two left people)
  × (number of people, excluding the three on the left)
  $= 4 \times 3 \times 2 \times 1 = 4! = 24$
- The event in which A is placed on the left is the number of people placed in the second, third, and right positions except A, so $3 \times 2 \times 1 = 3!$.
- Probability that A will be placed to the left is $3! / 4! = 6 / 24 = 0.25$.

# 5.2 Calculation of Probability

**[Ex 5.2.2] A company has four security guards (A, B, C, D). Each morning, two of these guards are randomly selected, one at the front gate and the other at the rear guard.**

- **Obtain total number of cases in which four people are placed at the front and rear gates and the number in which A is placed at the front gate.**
- **What is the probability that A will be placed at the front gate?**

**<Answer>**

- The number of elements in the sample space is:
  (number of people who can be placed at the front gate)
  × (number of people who can be placed in the rear except those in the front)
  = 4 × 3 = ${}_{4}P_{2} = 12$
- The number of elements where A will be placed at the front gate is ${}_{3}P_{1} = 3$
- Probability that A will be placed at the front gate one day is

$$\frac{{}_{3}P_{1}}{{}_{4}P_{2}} = \frac{3 \times 1}{4 \times 3} = \frac{1}{4}$$

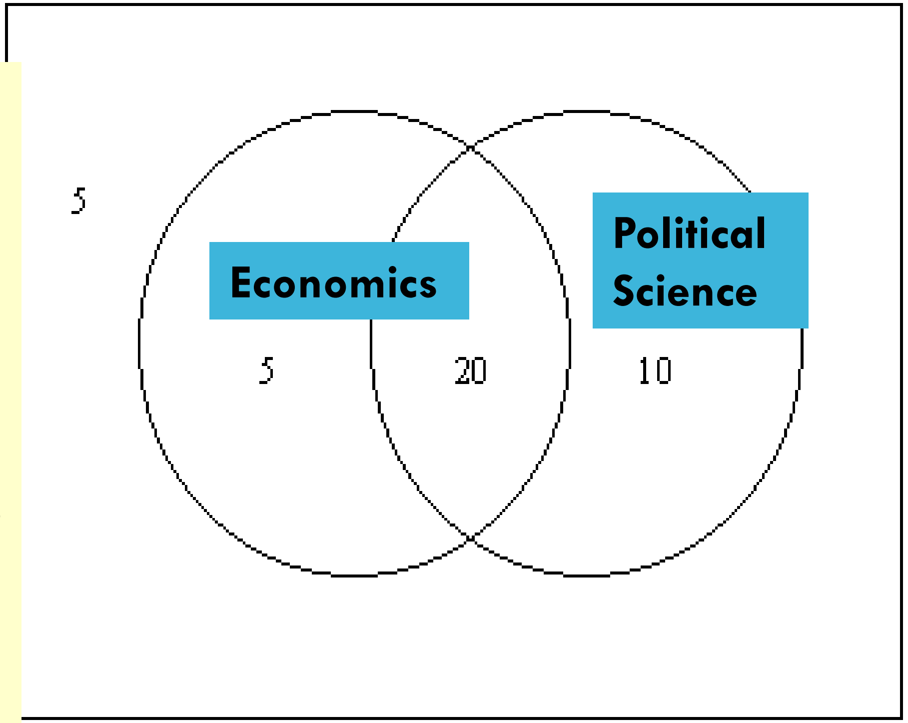
# 5.2 Calculation of Probability

[Example 5.2.3] Out of 40 sophomores in statistics department this semester, 25 students are taking economics(A), 30 students are taking political science(B) and 20 students are taking both subjects.

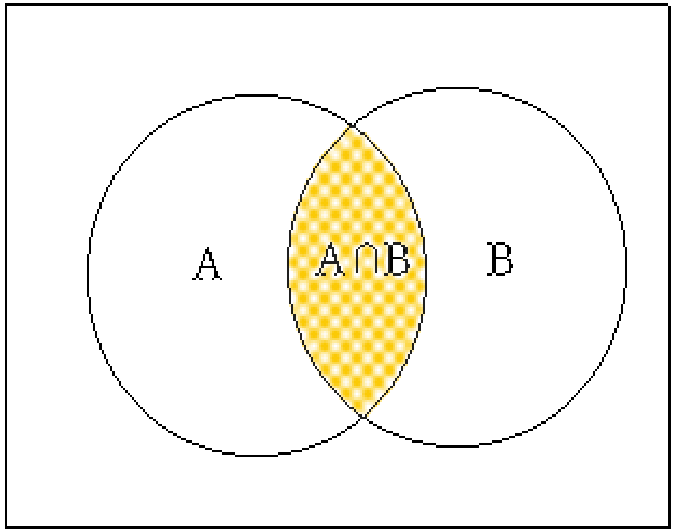
- When I meet one of the sophomores, what is the probability of this student taking either economics or political science?

<Answer>

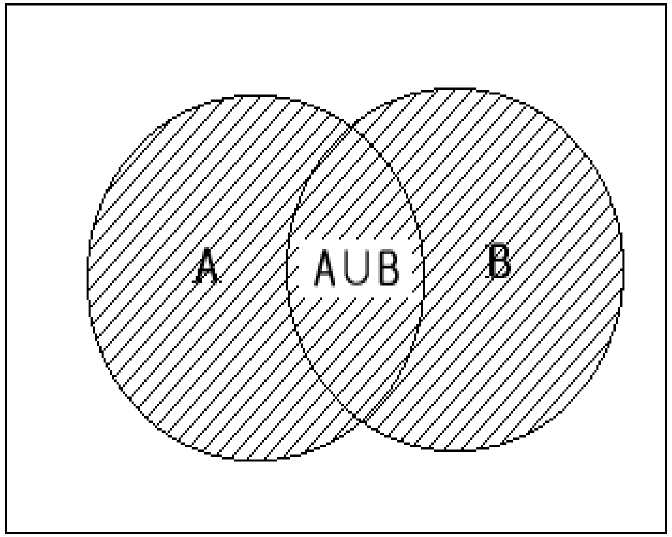
- Since there are 25 students who take economics and 20 students taking both courses, 25 - 20 = 5 students take only economics.
- Since there are 30 students who take political science, 30 - 20 = 10 students take only political science.
- The number of students taking economics or politics is 5 + 10 + 20 = 35.
- The probability of students taking economics or politics is 35 / 40.



# 5.2 Calculation of Probability




- Let's call the case of students taking economics A and the case of students taking political science B.
- Events that take both courses are marked as $A \cap B$ and are called an intersection event of A and B.
- The event in which a student takes a course in either economics or political science is marked as $A \cup B$ and is called a union event of A and B




- The probability of $P(A \cup B)$ can also be calculated as

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 25/40 + 30/40 - 20/40 = 35/40$$

- The probability of taking either economics or political science, $P(A \cup B)$, can be calculated by adding the probability of taking each course and then by subtracting the probability of taking both courses.

# 5.2 Calculation of Probability

## ❖ Addition Rule of Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If $A \cap B = \emptyset$, then the rule becomes as follows.

$$P(A \cup B) = P(A) + P(B)$$

In this case, the events A and B are called **mutually exclusive events.**

# 5.2 Calculation of Probability

**[Example 5.2.5]** Of the 30 sophomores in the Department of Statistics, there are 10 males and 20 females, one of males is from the province and five of females are from the province.

1) What is the probability that a student is from a province?
2) Among female students, what is the probability that a student is from a province?
3) Among province students, what's the probability of this student being a male?
4) What is the probability that a student is male and from Baku?

<Answer>

- It is convenient to organize the information into a table.

| | Baku | Province | Total |
|---|---|---|---|
| Male | _____ | 1 | 10 |
| Female | _____ | 5 | 20 |
| Total | _____ | _____ | 30 |

# 5.2 Calculation of Probability

- Calculate and insert the blanks along with the following.

| | Baku(B) | Province(C) | Total |
|---|---|---|---|
| Male(M) | 9 | 1 | 10 |
| Female(F) | 15 | 5 | 20 |
| Total | 24 | 6 | 30 |

1) P(C) = 6/30.
2) The probability that a student is from a province among female is 5/20.
   ⇨ P(C|F), conditional probability.
3) The probability of a male from a province origin is P(M|C) = 1/6.

# 5.2 Calculation of Probability

4) **The probability is $P(M \cap B)$ is 9/30.**
**Alternatively,**

$$P(M \cap B) = P(M)\,P(B|M) = (10/30) \times (9/10) = 9/30$$

$$P(B|M) = \frac{P(M \cap B)}{P(M)} = \frac{9/30}{10/30} = \frac{9}{10}$$

$$P(M \cap B) = P(S)\,P(M|B) = (24/30) \times (9/24)$$

# 5.2 Calculation of Probability

## ❖ Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad \text{if } P(B) \neq 0$$

## ❖ Multiplication Rule of Probability

$$P(A \cap B) = P(A)\,P(B|A)$$

If $P(B|A) = P(B)$, then $P(A \cap B) = P(A)\,P(B)$

** A and B are independent events

# 5.2 Calculation of Probability

[Ex 5.2.6] Tiger baseball team has probability to beat Lion team of 0.7 recently.

- What is the probabilities that the Tiger is winning both game on this evening double match? Assume that winning one game does not affect winning the next.

<Answer>

- Event that Tiger wins the first game is A and event that the Tiger wins the second game is B.
- Since A and B are independent of each other. the probability that the Tiger is winning both games is;

$$P(A \cap B) = P(A)\,P(B) = 0.7 \times 0.7 = 0.49$$

# 5.2 Calculation of Probability

[Ex 5.2.7] Table of 30 second-year students by gender and region.

- Are the events of male and Baku origin independent of each other?

| | Baku(B) | Province(C) | Total |
|---|---|---|---|
| Male(M) | 5 | 5 | 10 |
| Female(F) | 10 | 10 | 20 |
| Total | 15 | 15 | 30 |

<Answer>

$P(M \cap B) = 5/30$, $P(M) = 10/30$, $P(B) = 15/30$

⇨ $P(M \cap B) = P(M)\,P(B)$

⇨ Male and Baku are independent.

- Note that $P(M|S) = 5/15 = 1/3$, $P(M)=10/30$, therefore $P(M|S) = P(M)$.
- All items M and C, F and B, F and C are independent of each other => gender and region are independent.

# 5.2 Calculation of Probability

[Example 5.2.8] There is a box of six products, two of which are defective.

- What is the probability that at least one defective product will be found when three have been extracted for product testing? Assume that the product extracted without replacement.

<Answer>

- Probability of finding one defect in three products is $\frac{{}_4C_2 \times {}_2C_1}{{}_6C_3} = \frac{3}{5}$
- Probability of finding two other defective products is $\frac{{}_4C_1 \times {}_2C_2}{{}_6C_3} = \frac{1}{5}$
- Probability that at least one defect will be found is 3/5 + 1/5 = 4/5.
- Another way to obtain this probability is to use a complementary event.

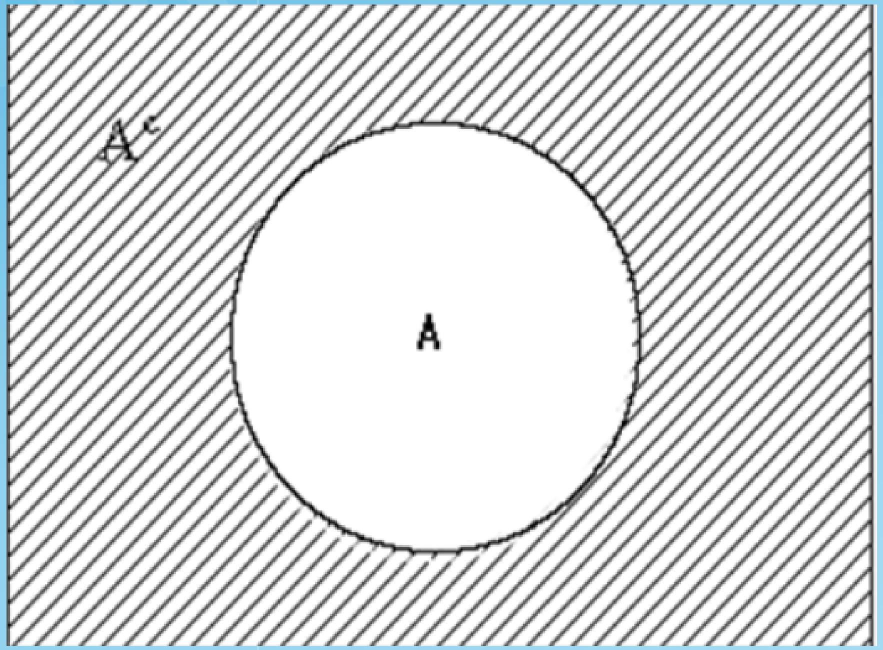
$$1 - \frac{{}_4C_3}{{}_6C_3} = 1 - \frac{4}{20} = \frac{4}{5}$$

# 5.2 Calculation of Probability

❖ **Probability of a complementary event**

If $A^C$ denotes a complementary event of A, then P($A^C$) can be calculated as follows.

$$P(A^C) = 1 - P(A)$$



# Thank you