Introduction to Statistics and Data Science using *eStat*

Chapter 5 Probability Distribution

5.3 Discrete Random Variable
– Poisson Distribution –

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5.3 Discrete Random Variable

5.3.2 Poisson Distribution

- Examples that are frequently observed around us.

  - Number of calls made to an office between 9 and 10 a.m. daily.
  - Number of one-day traffic accidents occurring at a certain intersection.
  - Number of defective spots per unit area of the fabric.

- Random variable that represents this 'occurrence of events per unit time or unit area' is Poisson random variable

⇨ Poisson distribution.

5.3 Discrete Random Variable

5.3.2 Poisson Distribution

- Poisson distribution examples.

  - Demand for a product sold every day at a certain store
  - The number of typos that occur on each page of a book
  - Number of accidents occurring during a week in a factory
  - Number of defectives per unit length of cloth
  - Number of radioactive particles released from radioactive materials

5.3 Discrete Random Variable

5.3.2 Poisson Distribution

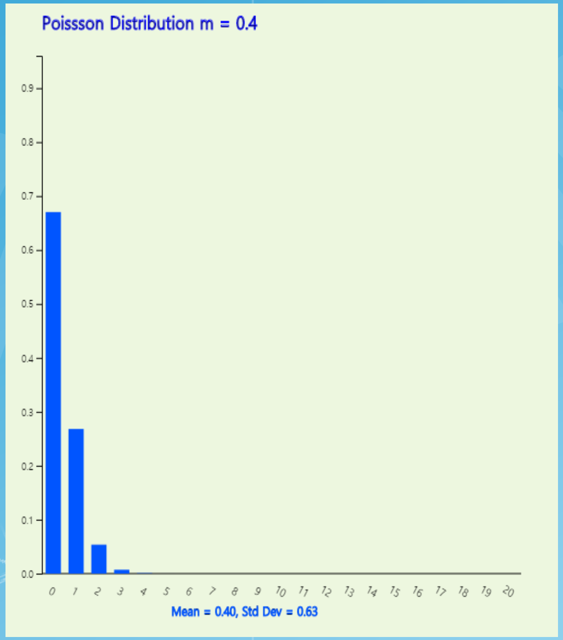
- The distribution of a Poisson random variable X = 'Occurrence of success event per unit time or unit area' is as follows when the average number of success is λ.

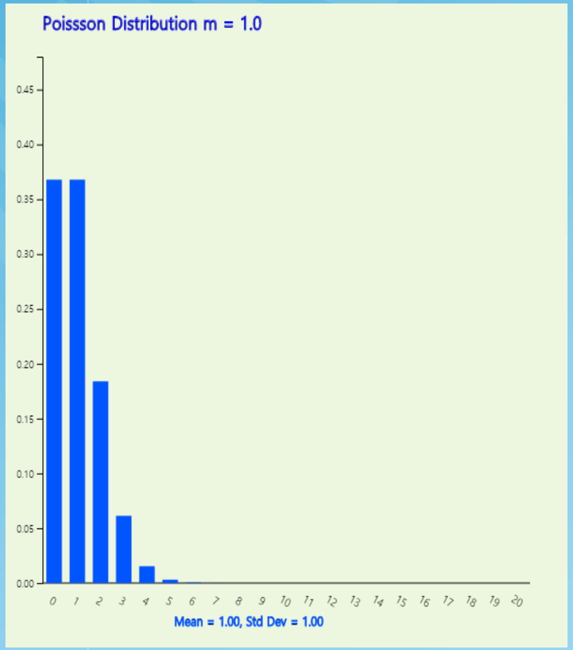
$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$$

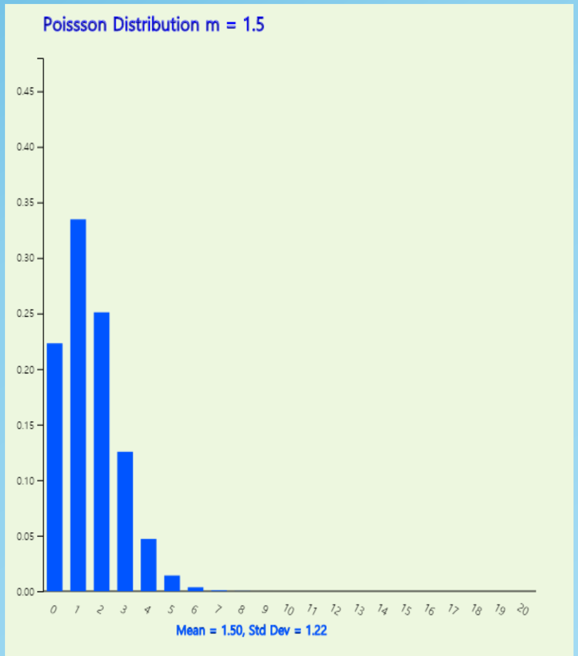
$$E(X) = \lambda, \quad V(X) = \lambda$$

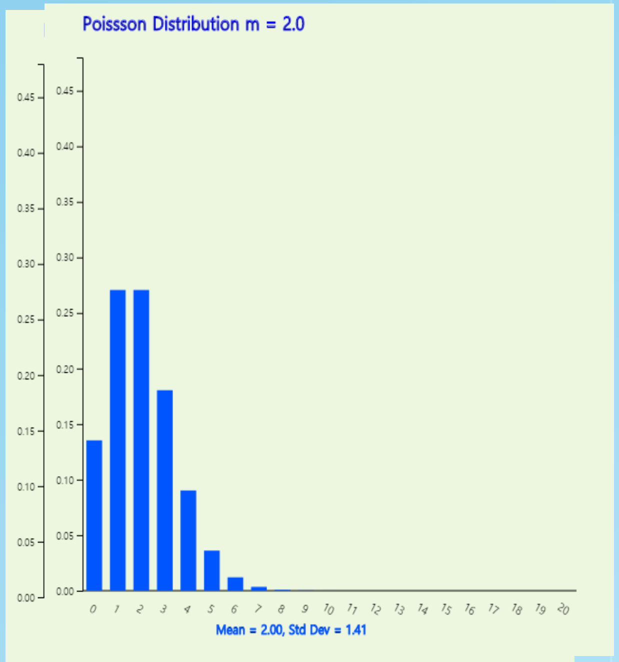
5.3 Discrete Random Variable

5.3.2 Poisson Distribution









5.3 Discrete Random Variable

5.3.2 Poisson Distribution

[Ex 5.3.8] Assume that cars per minute arriving at a highway toll gate during rush hour is the Poisson distribution of an average of five cars per one minute. One day, if you observe the toll gate for one minute during rush hour, calculate the following probabilities.

1) What is the probability that none of the cars will arrive?
2) What is the probability of five cars arriving?
3) What is the probability of more than two cars arriving?

Answer

- Let X be the Poisson random variable with $\lambda = 5$.

1) $P(X = 0) = f(0) = \dfrac{e^{-5}5^0}{0!} = 0.0067$

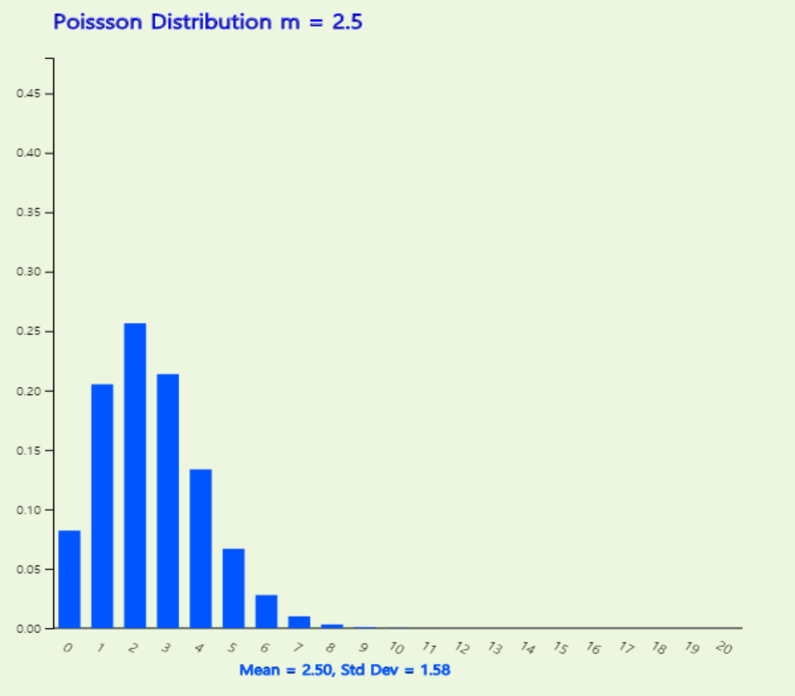
2) $P(X = 5) = f(5) = \dfrac{e^{-5}5^5}{5!} = 0.1755$

3) $P(X \geq 2) = 1 - P(X \leq 1) = 1 - P(X=0) - P(X=1)$
$= 1 - 0.0067 - 0.0337 = 0.9596$

5.3 Discrete Random Variable

[Ex 5.3.9] Assume that the average number of Typhoons passing through the southern part of the country is λ = 2.5 times per year. Check the following probabilities using 『eStatU』.

1) What is the probability that a Typhoon will pass once this year?
2) What is the probability that this year's Typhoon will pass twice or four times?
3) What is the probability that this year's Typhoon will pass more than once?



λ = 2.5			
x	P(X = x)	P(X x)	P(X x)
0	0.0821	0.0821	1.0000
1	0.2052	0.2873	0.9179
2	0.2565	0.5438	0.7127
3	0.2138	0.7576	0.4562
4	0.1336	0.8912	0.2424

5.3 Discrete Random Variable

〈Answer of Ex 5.3.9〉

1) $P(X=1) = 0.2052$.
2) $P(2 \le X \le 4)$ can be calculated as follows:

$$P(2 \le X \le 4) = P(X \le 4) - P(X \le 1) = 0.8912 - 0.2873 = 0.6039$$

This event can be calculated as $P(X=2) + P(X=3) + P(X=4)$.
3) If you use Table 5.3.7, $P(X \ge 2) = 0.7127$. Then the probability can be calculated by using the complementary event as follows:

$$P(X \ge 2) = 1 - P(X \le 1) = 1 - 0.2873 = 0.7127$$

Thank you