Introduction to Statistics and Data Science using *eStat*

Chapter 6 Sampling Distribution and Estimation

6.2 Sampling Distribution of Sample Means and Estimation of Population Mean

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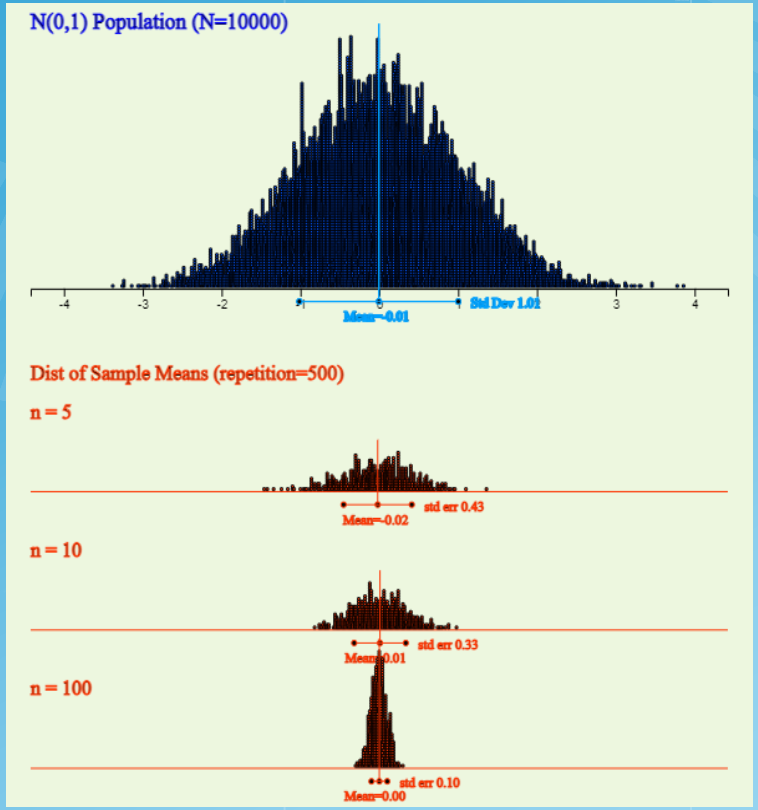
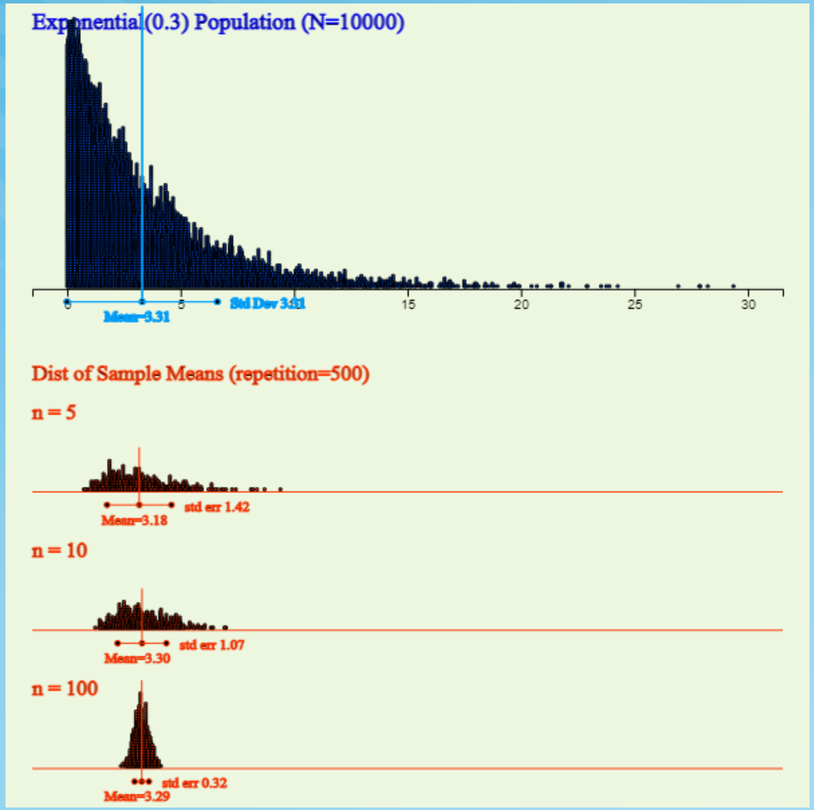
6.2.2 Estimation of Population Mean

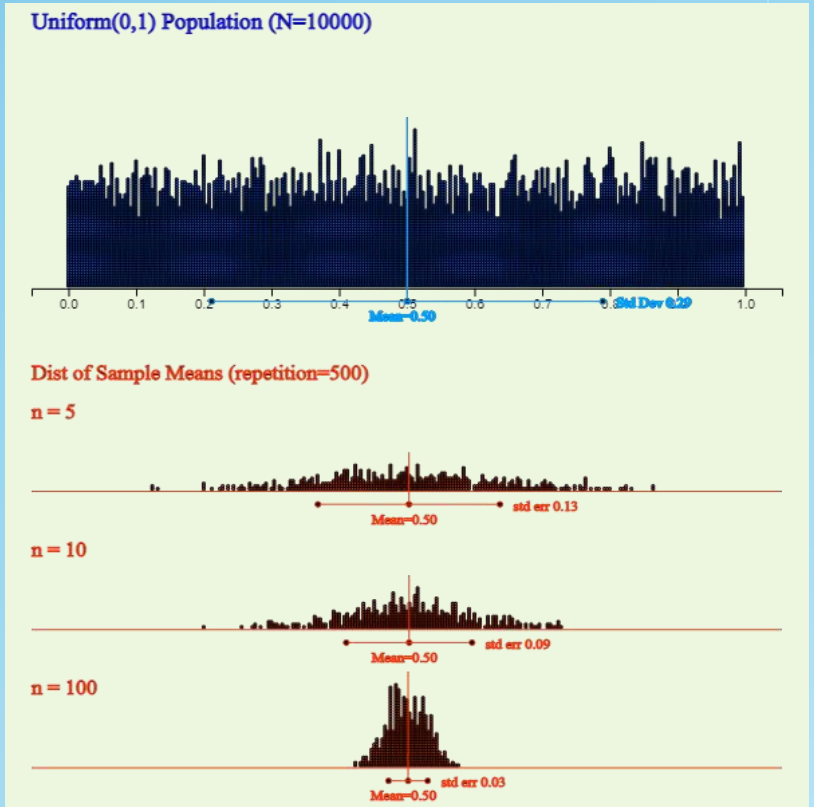
- Sample survey extracts only one set of samples from population.
  => sample mean of samples is an estimate of the population mean.
  => Is this sample mean is a good estimate for population mean?
  just only one set of samples?

- Sampling distribution of all possible sample means is the answer
  => if sample size is large enough, all possible sample means are clustered around the population mean in the form of a normal distribution.
  => Most of sample mean obtained from one set of samples is close to the population mean.
  => Even in worst case, error from population mean is not so significant.
  => The larger the sample size, the more concentrated the sampling distribution of the sample means, which reduces this error.

▪ Central Limit Theorem(CLT) - Simulation







6.2.2 Estimation of Population Mean

A. Point Estimation of Population Mean

- A value of observed sample mean is a point estimate of population mean.
- If the average of all possible sample statistics is equal to the population parameter, the sample statistic is called an unbiased estimator.
  - sample mean is an unbiased estimator of the population mean.
- When sample size grows, if sample statistic becomes closer to the population parameter, the sample statistic is called a consistent estimator.
  - sample mean is a consistent estimator of the population mean.
- If sample statistic has the least variance among several unbiased estimators, the sample statistic is called an effective estimator.
  - sample mean is an efficient estimator of the population mean.

6.2.2 Estimation of Population Mean

B. Interval Estimation of Population Mean – Known Population Variance

- 100(1-α)% Confidence Interval for Population Mean
  - Assume a population is a normal distribution and σ^2 is known.

$$\left[\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

- usually, $\alpha = 0.01\ or\ 0.05$ which is margin of error
  100(1- α)% is called a confidence level,

- 95% confidence interval $\bar{X} \pm 1.96\frac{\sigma}{\sqrt{n}}$, $z_{0.05/2} = z_{0.025} = 1.96$
- 99% confidence interval $\bar{X} \pm 2.575\frac{\sigma}{\sqrt{n}}$, $z_{0.01/2} = z_{0.005} = 2.575$

6.2.2 Estimation of Population Mean

B. Interval Estimation of Population Mean – Known Population Variance

- 『eStatU』Simulation of 95% confidence interval

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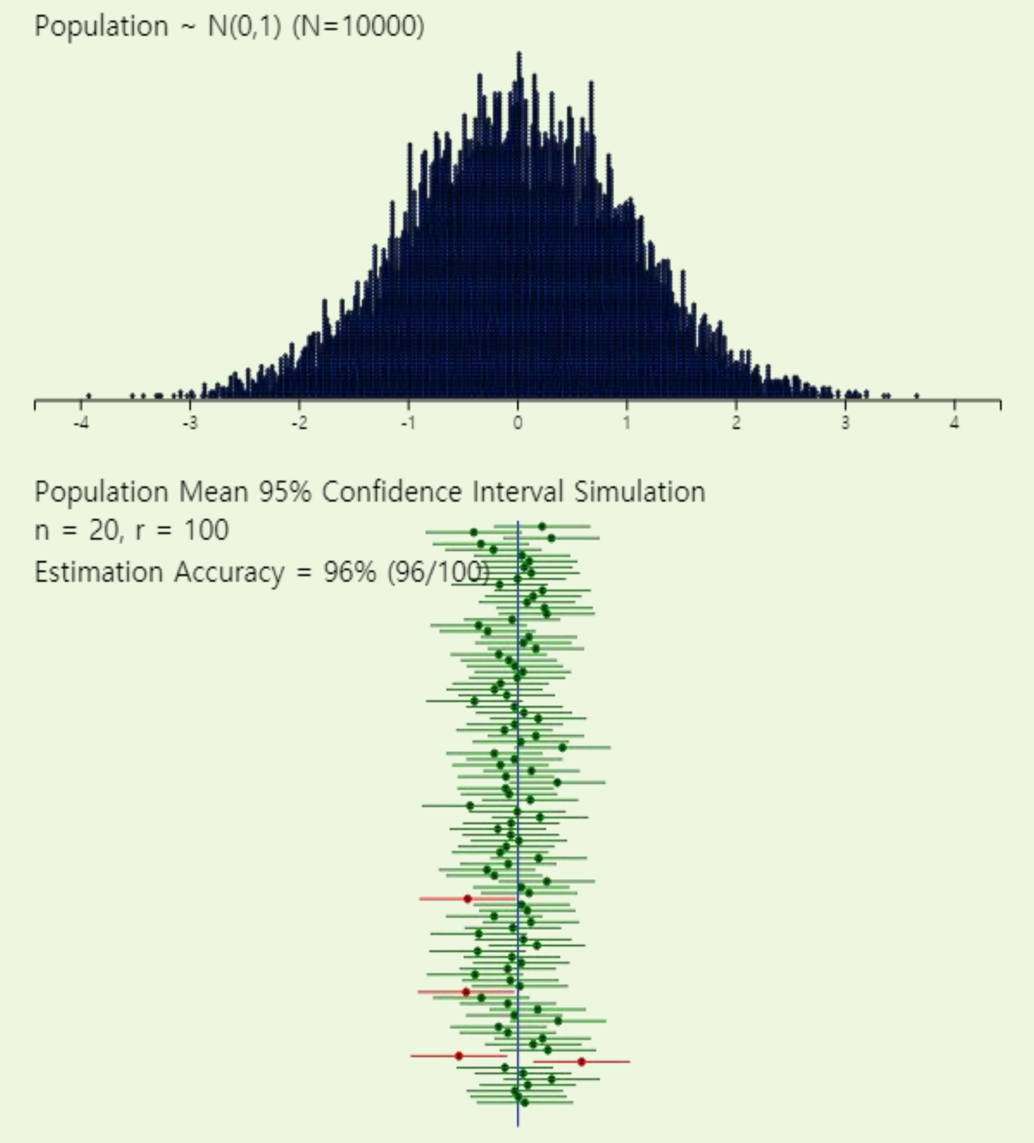

6.2.2 Estimation of Population Mean

[Ex 6.2.2] The average starting salary was 275 (unit man won) after a simple random sampling of 100 college graduates this year. Assume that the starting salary for all college graduates is a normal distribution and its standard deviation is 5 (unit man won).

1) Estimate the average starting salary of all college graduates.
2) Estimate a 95% confidence interval of the average starting salary of college graduates.
3) Estimate a 99% confidence interval of the average starting salary of college graduates. Compare the width of this interval to the 95% confidence interval?
4) If the sample size is 400, estimate a 95% confidence interval of the average starting salary for all college graduates. Compare the width of the interval to question 2)?

6.2.2 Estimation of Population Mean

<Answer of Ex 6.2.2>

1) Point estimation of the average starting salary is the sample mean which is 2.75 million won.

2) Since the 95% confidence interval implies α = 0.05, z value is as follows.

$$z_{\alpha/2} = z_{0.05/2} = z_{0.025} = 1.96$$

Therefore the 95% confidence interval is as follows.

$$\left(\overline{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} , \overline{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \right)$$

$\Leftrightarrow$ (275 − 1.96(5/10), 275 + 1.96(5/10))

$\Leftrightarrow$ (274.02, 275.98)

3) Since the 99% confidence interval implies α = 0.01, z value is as follows.

$$z_{\alpha/2} = z_{0.01/2} = z_{0.005} = 2.575$$

Therefore the 99% confidence interval is as follows.

(275 − 2.575(5/10) , 275 + 2.575(5/10))

$\Leftrightarrow$ (273.71, 276.29)

Therefore, if the confidence level is increasing, the width of the confidence interval becomes wider.

4) If the sample size is 400, the 95% confidence interval is as follows.

(275 − 1.96(5/20) , 275 + 1.96(5/20))

$\Leftrightarrow$ (274.51, 275.49)

Therefore, if the sample size is increasing, the width of the confidence interval becomes narrower which is an accurate estimation.

6.2.2 Estimation of Population Mean

B. Interval Estimation of Population Mean – Unknown Population Variance

- 100(1-α)% Confidence Interval for Population Mean
  - Assume a population is a normal distribution and σ^2 is known.
  - S is sample standard deviation

$$\left[\bar{X} - t_{n-1;\,\alpha/2}\frac{S}{\sqrt{n}},\ \bar{X} + t_{n-1;\,\alpha/2}\frac{S}{\sqrt{n}}\right]$$

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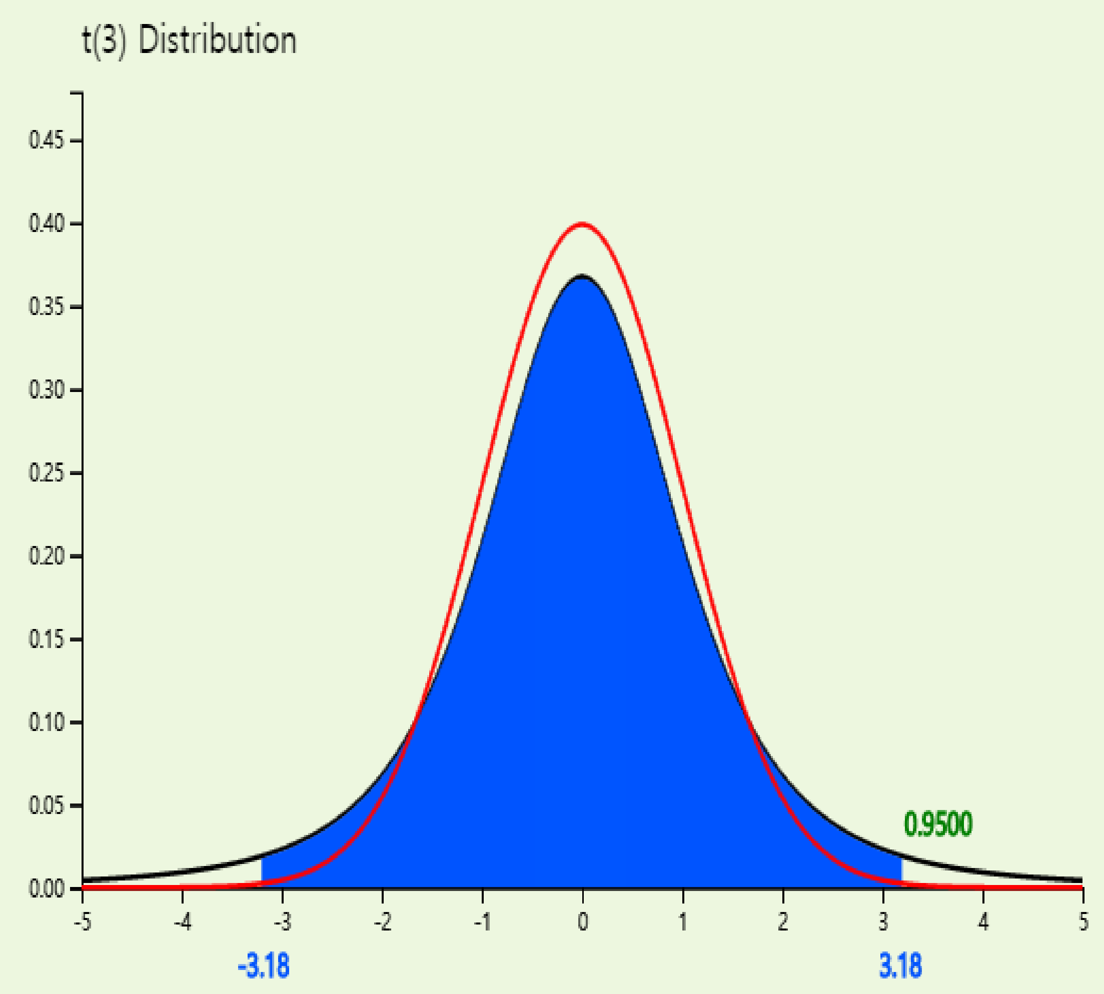
- usually $\alpha = 0.01\ or\ 0.05$ which is margin of error
  100(1- α)% is called a confidence level,

6.2.2 Estimation of Population Mean

- t distribution was studied by a statistician W. S. Gosset in Ireland.
- t distribution is not just a single distribution, but it is a family of distributions under a parameter called a degree of freedom, 1,2, … , 30, … and denoted as $t_1, t_2, \dots, t_{30}, \dots$
- The shape of the t distribution is symmetrical about zero (y axis), similar to the standard normal distribution, but it has a tail that is flat and longer than the standard normal distribution.



6.2.2 Estimation of Population Mean

[Ex 6.2.3] Suppose we do not know the population variance In Example 6.2.2. If the sample size is 25 and the sample standard deviation is 5 (unit man won), estimate the mean of the starting salary of college graduates at 95% confidence level.

<Answer>

- Since we do not know the population variance, t distribution should be used for interval estimation of the population mean.
- Since $t_{n-1;\,\alpha/2} = t_{25-1;0.05/2} = t_{24;0.025} = 2.0639$, the 95% confidence interval of the population mean is as follows:

$$[\overline{X} - t_{n-1;\,\alpha/2} \frac{S}{\sqrt{n}} \;, \overline{X} + t_{n-1;\,\alpha/2} \frac{S}{\sqrt{n}}]$$

⇔ (275 - 2.0639 (5/5) , 275 + 2.0639 (5/5))

⇔ (272.9361, 277.0639)

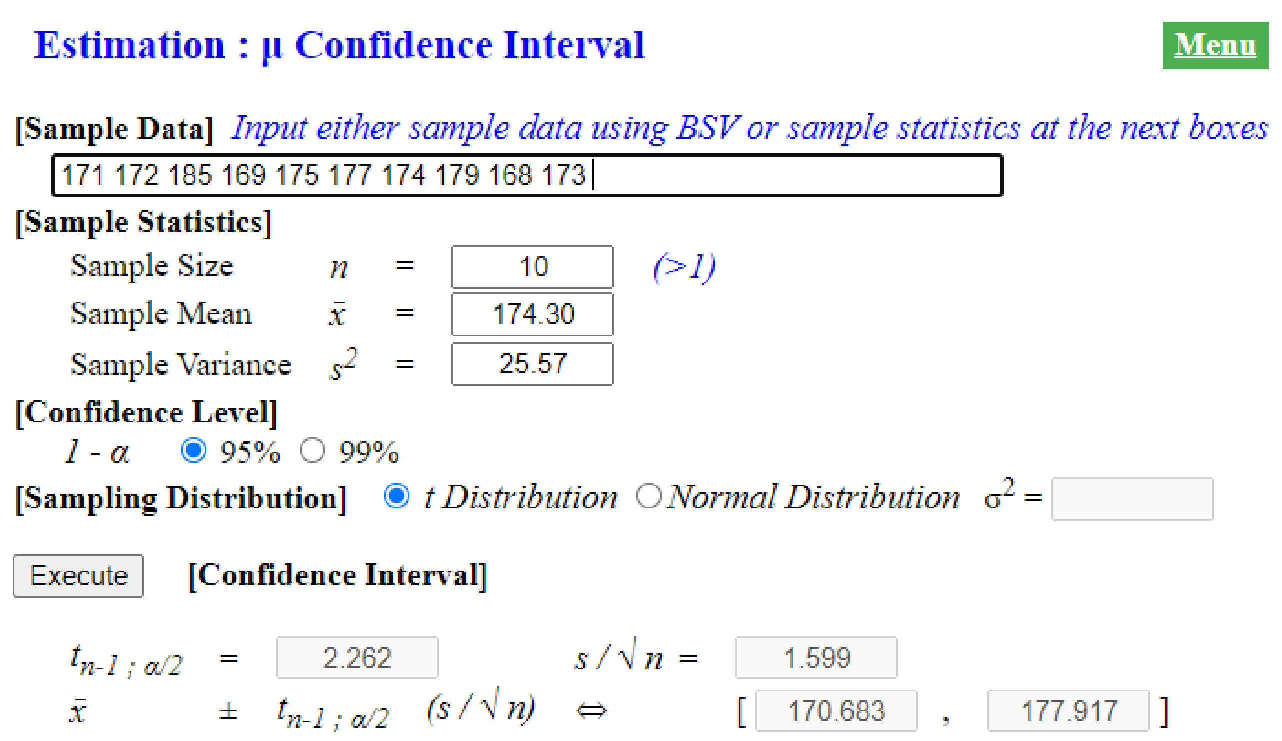
6.2.2 Estimation of Population Mean

[Example 6.2.4] The following data shows a simple random sampling of 10 new male students this year to investigate the heights of university students. Use 『eStatU』 to make a 95% confidence interval of the height of college freshmen.

171 172 185 169 175 177 174 179 168 173

<Answer>

- Click ‘Estimation: μ Confidence Interval’ at the 『eStatU』 and enter data at the [Sample Data] box.
- Then the confidence intervals [170.68, 177.92] are calculated using the t distribution.



Thank you