Introduction to Statistics and Data Science using *eStat*

Chapter 6 Sampling Distribution and Estimation

6.3 Sampling Distribution of Sample Variances and Estimation of Population Variance

Jung Jin Lee
Professor of Soongsil University, Korea
Visiting Professor of ADA University, Azerbaijan

6.3.1 Sampling Distribution of Sample Variances

[Ex 6.3.1] Let's consider the data again in Example 6.2.1 which is the number of years of service for the five salespeople.

6, 2, 4, 8, 10

1) Calculate the population variance.

2) Find all possible samples of size 2 with replacement and calculate the sample variance of each sample. In addition, calculate the average and variance of all of these sample variances and compare them to the population variance.

3) Find a frequency distribution of all possible sample variances and draw a bar chart.

<Answer>

1) The population mean is $\mu = 6$ and variance is 8.

6.3.1 Sampling Distribution of Sample Variances

<Answer of Ex 6.2.1>

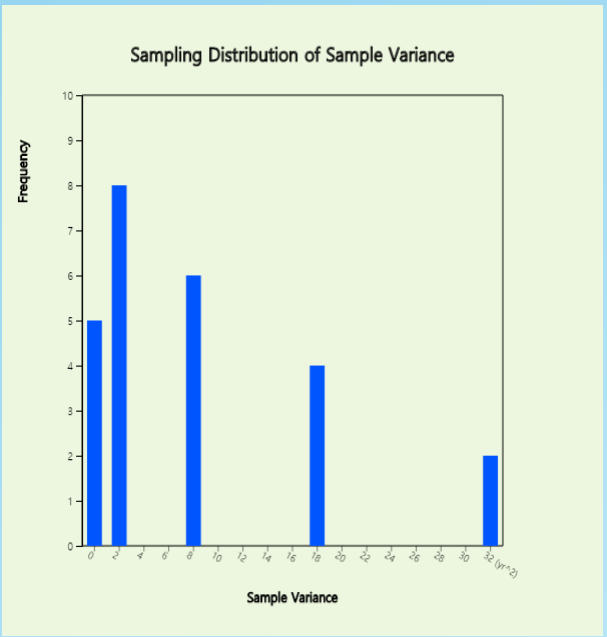
2) All possible samples of size 2 with replacement from the population and the sample variance of each sample

Sample	s^2	Sample	s^2	Sample	s^2	Sample	s^2	Sample	s^2
2,2	0	4,2	2	6,2	8	8,2	18	10,2	32
2,4	2	4,4	0	6,4	2	8,4	8	10,4	18
2,6	8	4,6	2	6,6	0	8,6	2	10,6	8
2,8	18	4,8	8	6,8	2	8,8	0	10,8	2
2,10	32	4,10	18	6,10	8	8,10	2	10,10	0

- **the average of all possible sample variances is the same as the population variance which means the sample variance is the unbiased estimate of the population variance.**

- **Sampling distribution of sample variance**

Sample variance	Frequency	Relative frequency
0	5	0.20
2	8	0.32
8	6	0.24
18	4	0.16
32	2	0.08
	25	1.00



6.3.1 Sampling Distribution of Sample Variances

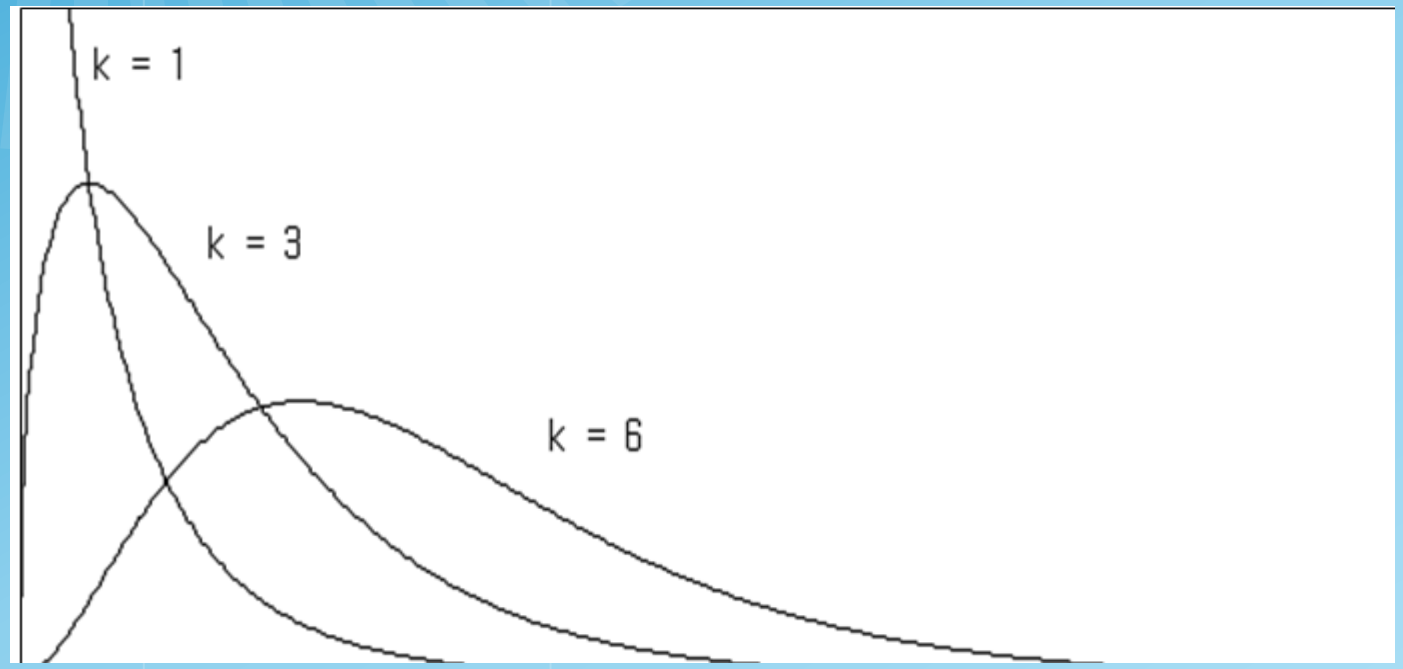
- **Sampling Distribution of Sample Variance**

When the population is normally distributed and the sample of size n is selected randomly with replacement, the distribution of all sample variances multiplied by a specific constant follows the chi-square distribution with n-1 degrees of freedom as follows:

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

6.3.1 Sampling Distribution of Sample Variances

- **Chi-square distribution is a family of distributions depending on the degree of freedom, such as χ^2_1, χ^2_2, … , χ^2_{30}, … etc.**
- **Chi-square distribution is an asymmetrical distribution. If the degree of freedom is small, it is much skewed to the right.**



Thank you