Introduction to Statistics and Data Science using *eStat*

Chapter 6 Sampling Distribution and Estimation

6.3 Sampling Distribution of Sample Variances and Estimation of Population Variance

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6.3.2 Estimation of Population Variance

- **Examples of estimating population variance.**

- Two companies supply bolts to an automaker. Bolts are defective even if they are too large or too small in diameter. The automaker wants to recognize the variance of bolt diameters supplied by each bolt company and use them as data for decision making.

- To evaluate the difficulty of the college entrance exam conducted this year, the variance of the exam scores is calculated and compared with the variance of the exam score of previous year.

6.3.2 Estimation of Population Variance

- **Point estimation of the population variance σ^2**
  ⇒ Sample variance S^2 (S^2 is an unbiased estimator of σ^2)

- **Point estimation of the population standard deviation σ**
  ⇒ Sample standard deviation S (S is not an unbiased estimator of σ)

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6.3.2 Estimation of Population Variance

- 100(1-α)% Confidence interval of the population variance σ^2
  - Assume population is a normal distribution

$$\left[\frac{(n-1)S^2}{\chi^2_{n-1;\,\alpha/2}} , \ \frac{(n-1)S^2}{\chi^2_{n-1;1-\alpha/2}} \right]$$

S^2 is sample variance

- 100(1-α)% Confidence interval of the population standard deviation σ
  - Assume population is a normal distribution and large sample

$$\left[\sqrt{\frac{(n-1)S^2}{\chi^2_{n-1;\,\alpha/2}}} , \sqrt{\frac{(n-1)S^2}{\chi^2_{n-1;1-\alpha/2}}} \right]$$

S is sample standard deviation

6.3.2 Estimation of Population Variance

[Ex 6.3.2] A survey for the starting salary of 25 college graduates this year shows the sample standard deviation is 5 (1000$). Find point estimation and 95% confidence interval of the population variance and the population standard deviation. Assume that the population is normally distributed.

<Answer>

- The point estimate of the population variance is $s^2 = 25$.
- The point estimate of the population standard deviation is $S = 5$.
- The 95% confidence interval of the population variance is as follows:

$$\left[\frac{(25-1)S^2}{\chi^2_{25-1;0.05/2}} , \frac{(25-1)S^2}{\chi^2_{25-1;1-0.05/2}} \right]$$

$$\Leftrightarrow \left[\frac{(25-1)5^2}{39.364} , \frac{(25-1)5^2}{12.401} \right]$$

$$\Leftrightarrow [\ 15.242\ ,\ 48.383\ ]$$

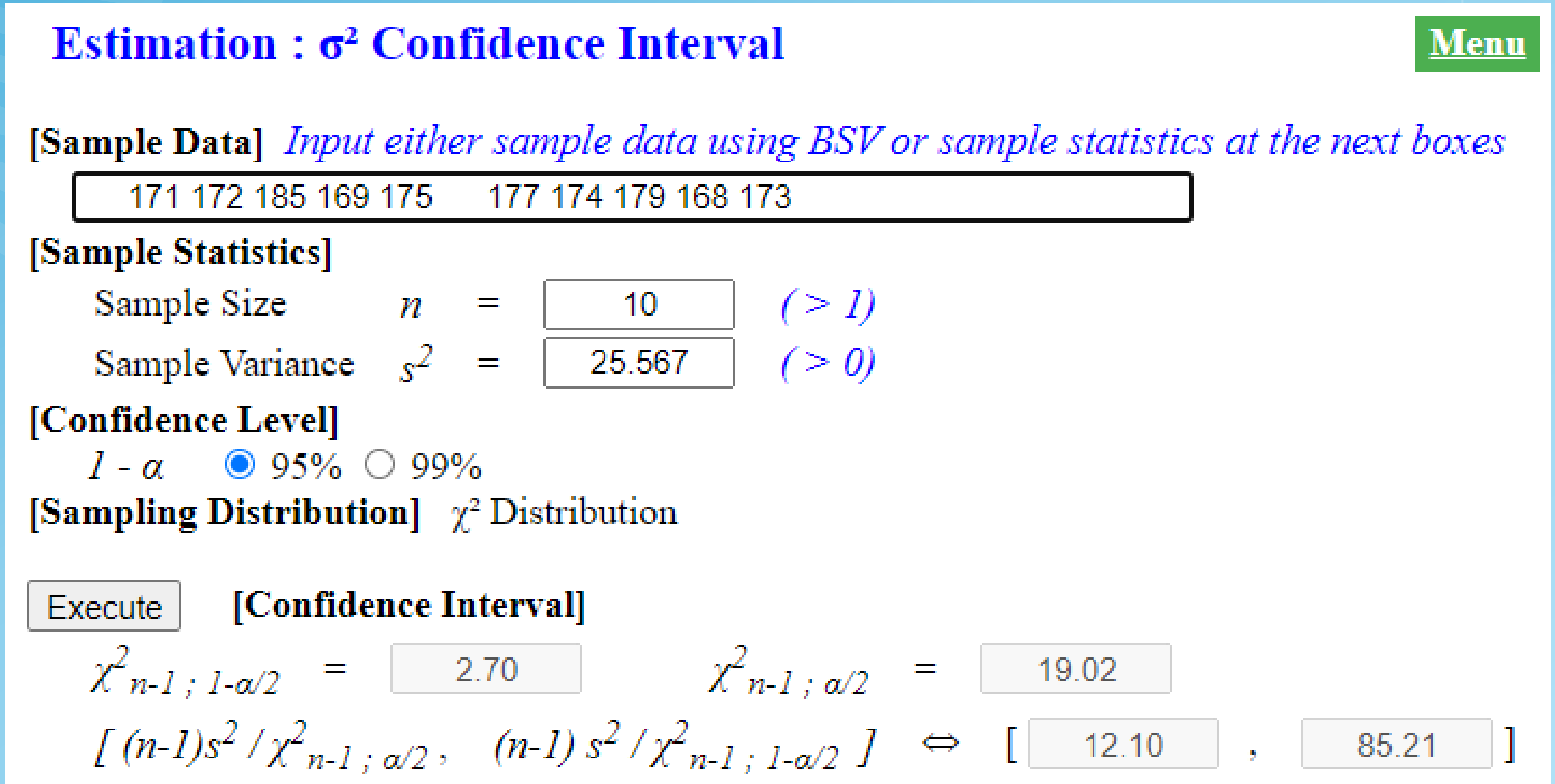
- The 95% confidence interval of the population standard deviation is $[\sqrt{15.242}, \sqrt{48.383}] \Leftrightarrow [3.904\ ,\ 6.956]$

6.3.2 Estimation of Population Variance

[Ex 6.3.3]The height data of 10 male freshman samples is as follows.

171 172 185 169 175
177 174 179 168 173

Use 『eStatU』 to make a 95% interval estimate of the population variance.



Thank you