Introduction to Statistics and Data Science using *eStat*

Chapter 6 Sampling Distribution and Estimation

6.4 Sampling Distribution of Sample Proportions and Estimation of Population Proportion

Jung Jin Lee
Professor of Soongsil University, Korea
Visiting Professor of ADA University, Azerbaijan

6.4.2 Estimation of Population Proportion

- Examples to estimate the proportion of unknown populations.

- What is approval rating of a particular political party in this year's election?
- What percentage of the nation's current unemployment rate is?
- What percentage of defective products do we have here when we import 10,000 car accessories?

- Sampling distribution of sample proportions

$$\hat{p} \sim N\left(p, \frac{p(1-p)}{n}\right)$$

6.4.2 Estimation of Population Proportion

- **Point estimate of the population proportion p ⇒ The sample proportion .**
  The sample proportion ($\hat{p}$) is an unbiased, efficient and consistent estimator of the population proportion p and the estimate of the standard error of $\hat{p}$ is $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

- **Interval estimation of the population proportion**
  If the population proportion is p, 100(1-α)% confidence interval of p when the sample size n is large is as follows:

$$\left[\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$

  Criteria of large sample size n are $n\hat{p} > 5$, $n(1-\hat{p}) > 5$.

6.4.2 Estimation of Population Proportion

[Example 6.4.3] A student running for president of a university had a simple survey of 200 students to find out his approval ratings, and found that 120 students supported him. Estimate the population's approval rating, and find a 95% confidence interval. Check the interval estimation using『eStatU』

<Answer>

- Estimation of the population approval rating is the sample proportion.

$$\hat{p} = \frac{120}{200} = 0.6$$

- The 95% confidence interval is as follows.

$$\left[\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$

$$\Leftrightarrow \left[0.6-1.96\sqrt{\frac{0.6(1-0.6)}{200}},\ 0.6+1.96\sqrt{\frac{0.6(1-0.6)}{200}}\right]$$

$$\Leftrightarrow [0.532\ ,\ 0.668]$$

Estimation : p Confidence Interval

[Sample Data]

Sample Size n = 200

Sample Proportion $\hat{p}$ = 0.6 $0 < \hat{p} < 1$

[Confidence Level]

$1-\alpha$ ◉ 95% ○ 99%

[Sampling Distribution] Normal Distribution

Execute **[Confidence Interval]**

$z_{\alpha/2}$ = 1.9600 $\sqrt{\hat{p}(1-\hat{p})/n}$ = 0.0346

$\hat{p} \pm z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n} \Leftrightarrow$ [0.5321 , 0.6679]

Thank you