Introduction to Statistics and Data Science using *eStat*

Chapter 6 Sampling Distribution and Estimation

6.6 Application of Sampling Distribution: Quality Control

Jung Jin Lee
Professor of Soongsil University, Korea
Visiting Professor of ADA University, Azerbaijan

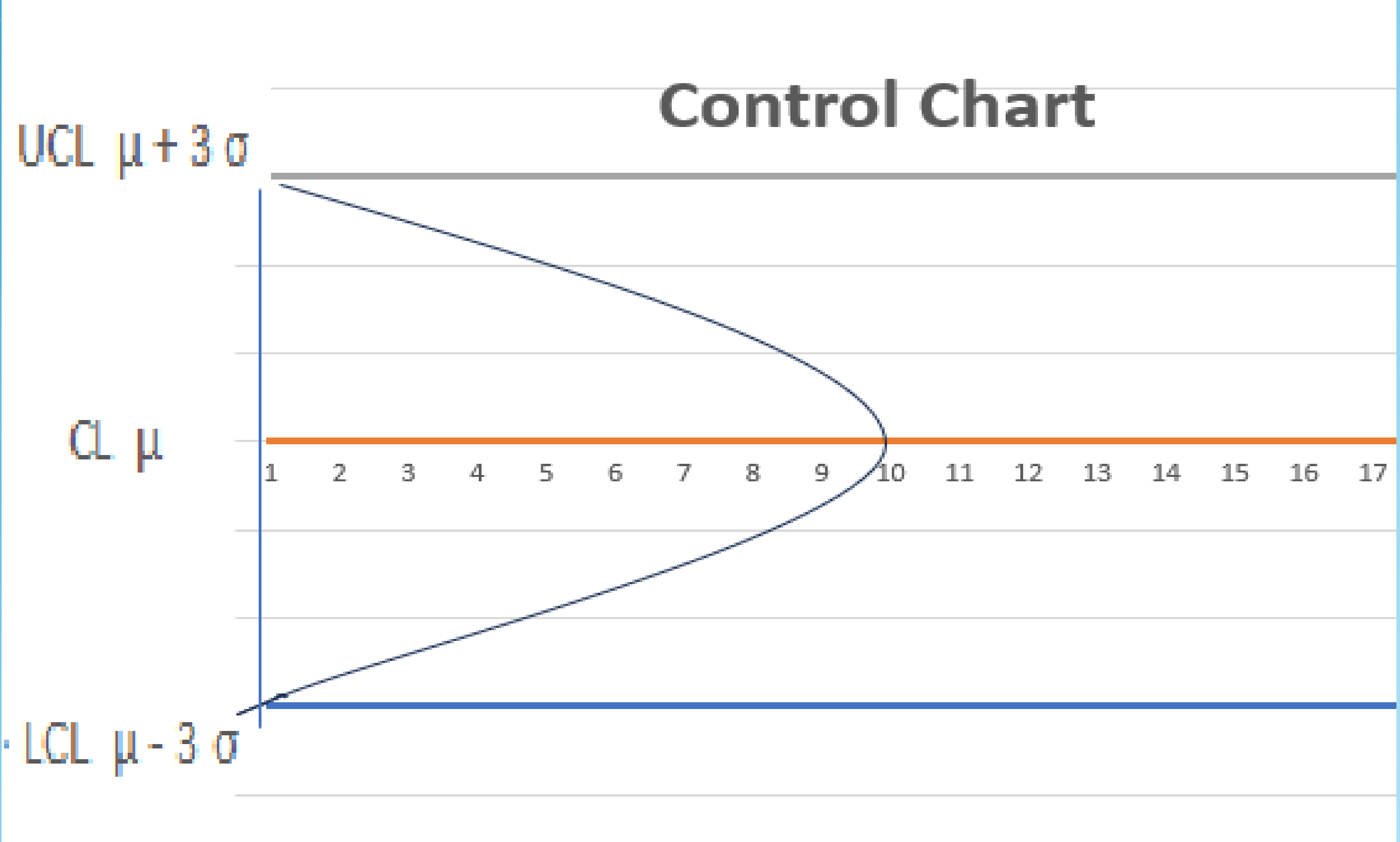
6.6 Application fo Sampling Distribution: Quality Control

- **When producing a product in general, there is a variation in the quality characteristic values even if it is produced under the same working conditions. The causes of the variation can be divided into chance causes and assignable causes**
- **If the quality characteristic value exceeds a certain limit, it is regarded as an explainable cause, and the cause is investigated and an action is taken. It is called a statistical quality control.**
- **One of the statistical tools widely used for this purpose is the control chart which was first introduced by W.A Shewart in 1924.**

6.6 Application fo Sampling Distribution: Quality Control

- **Control chart is a diagram to manage changes in quality characteristic values. There is an upper control limit (UCL) at the top, a center line (CL) in between, and a lower control limit (LCL) at the bottom. The characteristic value of an inspected sample in each time are plotted as a line graph.**
- **When the quality characteristic values do not exceed the control limit lines and are not related to each other, the process is said to be under control. If the quality characteristic value is outside the control limits or, if the characteristic values are related each other, the process is said to be out of control.**

6.6 Application fo Sampling Distribution: Quality Control



6.6.1 Control Chart by Variable

- **Sample mean control chart ($\bar{X}$ chart)**
  - **based on the sampling distribution of sample means**

    $\bar{X} \sim Normal\ (\mu, \sigma^2/n)$

  - **99.74% of sample means are in this interval**

$$[\mu - 3\frac{\sigma}{\sqrt{n}},\ \mu + 3\frac{\sigma}{\sqrt{n}}]$$

(

- **Upper Control Limit**
- **Center Line**
- **Lower Control Limit**

$$UCL_{\bar{X}} = \mu + 3\frac{\sigma}{\sqrt{n}}$$

$$CL_{\bar{X}} = \mu$$

$$LCL_{\bar{X}} = \mu - 3\frac{\sigma}{\sqrt{n}}$$

6.6.1 Control Chart by Variable

Let $\overline{x}_1, \overline{x}_2, \cdots, \overline{x}_k$ be the sample means obtained by selecting k samples of size n, and let $\overline{\overline{x}}$ be the average of these sample means. Let $R_1, R_2, \ldots, R_k$ be the sample range (maximum - minimum) of each sample, and let $\overline{R}$ be the average of these ranges as following:

$$\overline{\overline{x}} = \frac{\sum_{i=1}^{k} \overline{x}_i}{k}, \quad \overline{R} = \frac{\sum_{i=1}^{k} R_i}{k}$$

Table 6.6.1 Constants which are used in control charts

sample size (n)	A_2	D_3	D_4	d_2
2	1.880	0	3.267	1.128
3	1.023	0	2.574	1.693
4	0.729	0	2.282	2.059
5	0.577	0	2.114	2.326

$$\mathrm{UCL}_{\overline{x}} = \overline{\overline{x}} + A_2 \overline{R}$$
$$\mathrm{CL}_{\overline{x}} = \overline{\overline{x}}$$
$$\mathrm{LCL}_{\overline{x}} = \overline{\overline{x}} - A_2 \overline{R}$$

$$\hat{\sigma} = \frac{\overline{R}}{d_2}$$

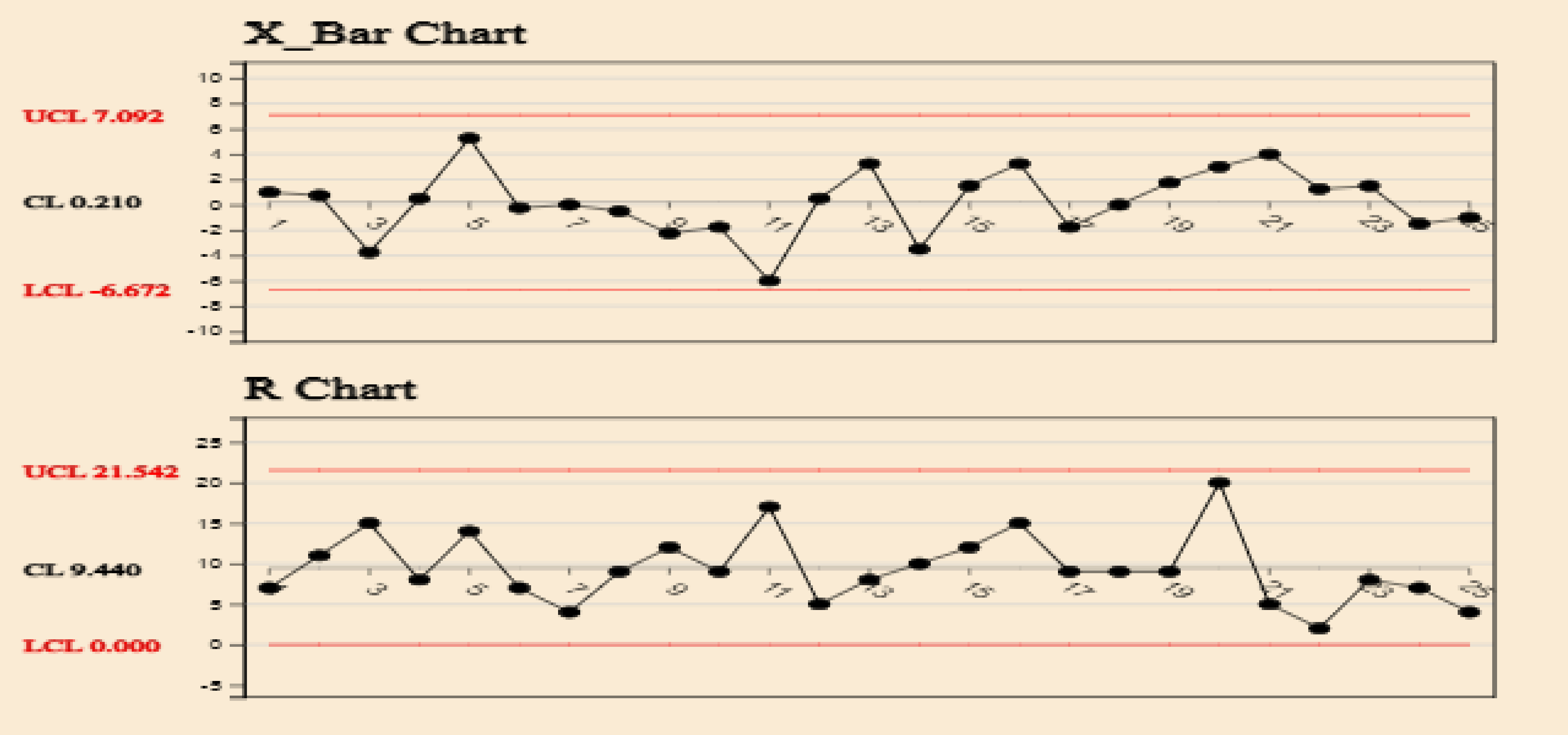
6.6.1 Control Chart by Variable

- **Sample range control chart (R chart)**

$$\mathrm{UCL_R} = \overline{R} + 3\hat{\sigma}_R = \overline{R} + 3d_3\frac{\overline{R}}{d_2}$$
$$\mathrm{CL_R} = \overline{R}$$
$$\mathrm{LCL_R} = \overline{R} - 3\hat{\sigma}_R = \overline{R} - 3d_3\frac{\overline{R}}{d_2}$$

$$\mathrm{UCL_R} = \overline{R}\,D_4$$
$$\mathrm{CL_R} = \overline{R}$$
$$\mathrm{LCL_R} = \overline{R}\,D_3$$

6.6.1 Control Chart by Variable



6.6.1 Control Chart by Attribute

- **Sample defective rate control chart (p chart)**

  - based on the sampling distribution of sample proportions

  $(\hat{p}) \sim Normal$ **(p, p(1-p)/n)**

  - **99.74% of sample proportions are in this interval**

$$[\ p-3\sqrt{\frac{p(1-p)}{n}}\ ,\ \ p+3\sqrt{\frac{p(1-p)}{n}}\ ]$$

- **Upper Control Limit**
- **Center Line**
- **Lower Control Limit**

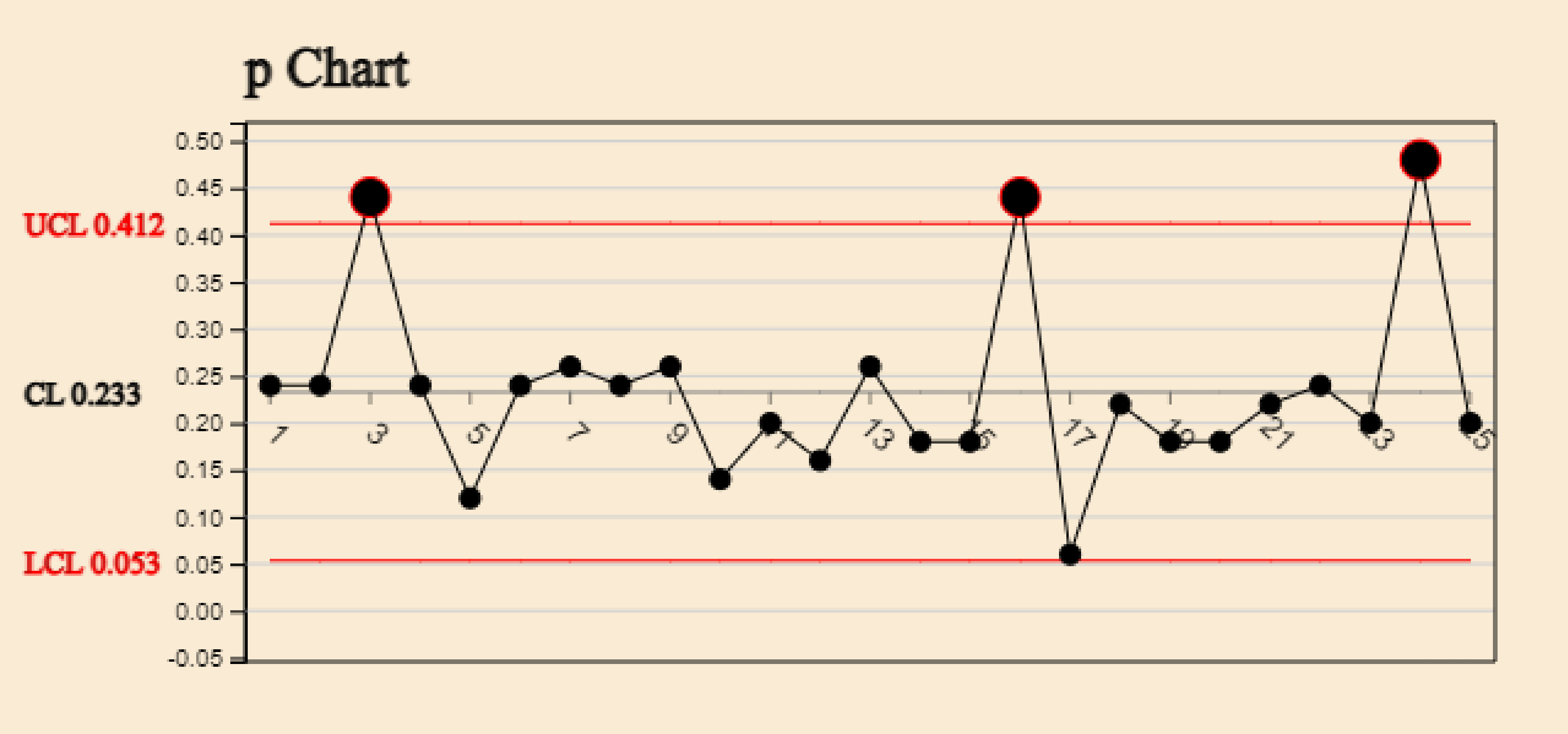
$$UCL_p = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n}$$

$$CL_p = \bar{p}$$

$$LCL_p = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n}$$

$$\bar{p} = \frac{number\ of\ defective\ products\ in\ all\ samples}{number\ of\ products\ in\ all\ samples}$$

6.6.1 Control Chart by Attribute



Thank you