Introduction to Statistics and Data Science using *eStat*

## Chapter 7 Testing Hypothesis for Single Population

# 7.4 Testing Hypothesis with $\alpha$ and $\beta$ simultaneously

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# 7.4 Testing Hypothesis with $\alpha$ and $\beta$ simultaneously

- Testing hypothesis so far is a conservative decision−making method.
  ⇨ critical value that reduces the probability of type one error $\alpha$
  (error that rejects the null hypothesis even though it is true).
  ⇨ keep $H_0$ unless there is sufficient evidence of $H_1$ which is risky.
  ⇨ probability of type two error ($\beta$) was not considered.

- Sometimes it is unclear which should be $H_0$ and which one should be $H_1$.
  ⇨ both $\alpha$ and $\beta$ are important and should be considered simultaneously.
  ⇨ If the analyst can determine the sample size,
  testing hypothesis with $\alpha$ and $\beta$ can be performed.

# 7.4 Testing Hypothesis with $\alpha$ and $\beta$ simultaneously

## 7.4.1 $\beta$ and the power of test

[Example 7.4.1] In [Example 7.1.1], calculate the probability of the type 2 error $\beta$ if the significance level is 5%. Check this result using 『eStatU』.

〈Answer〉

- Hypothesis is $H_0$ : $\mu$ = 1500, $H_1$ : $\mu$ = 1600,
- Population standard deviation is assumed $\sigma$ = 200, and $n$ =30.
- Decision rule is as follows if $\alpha$ is 5%.

  'If $\bar{X} < 1500 + (1.645)\sqrt{\frac{200^2}{30}} = 1560.06$, reject $H_0$'

- Type 2 error (probability of $H_0$ is true when $H_1$ is true) is calculated as:

  $\beta = P(\bar{X} < 1560.06 \mid H_1 \text{ is true})$

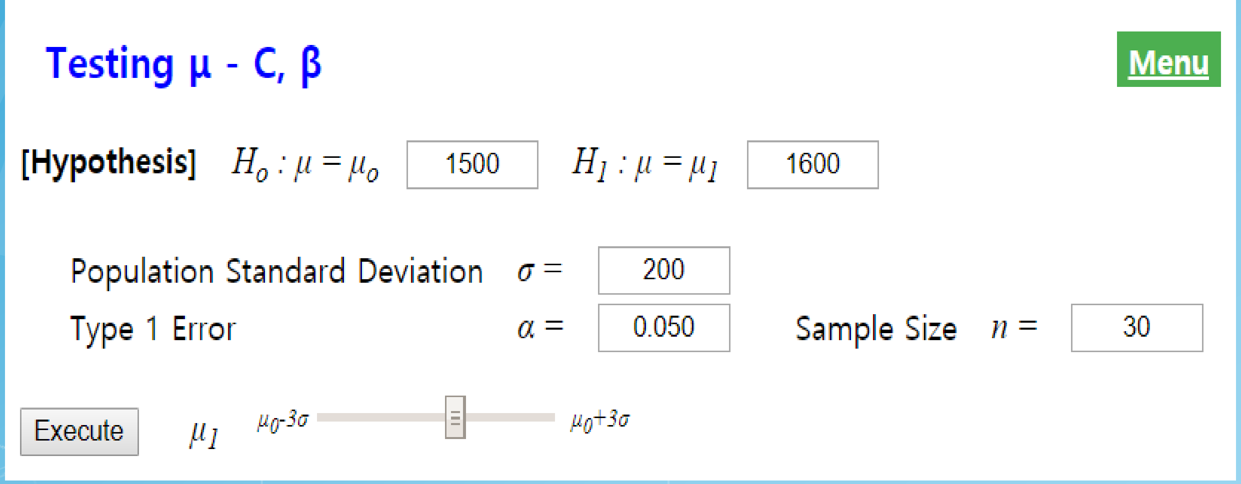
  $= P((\bar{X} - 1600) / \sqrt{\frac{200^2}{30}} < (1560.06 - 1600) / \sqrt{\frac{200^2}{30}})$

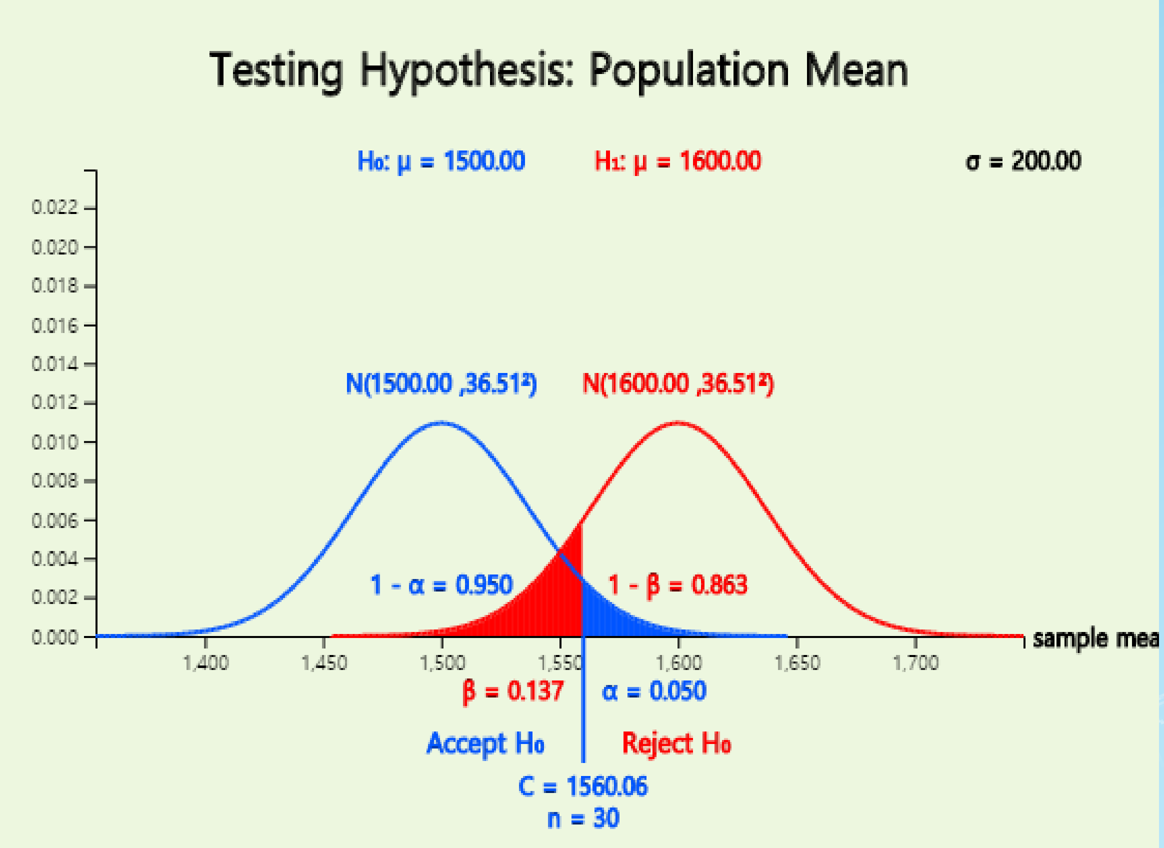
  $= P(Z < -1.09) = 0.137$

# 7.4 Testing Hypothesis with $\alpha$ and $\beta$ simultaneously

⟨Answer of Ex 7.4.1⟩ Select 'Testing $\mu - C,\ \beta$' at 『eStatU』 menu.

- Enter $\mu_0 = 1500$ , $\mu_1 = 1600$, $\sigma$ = 200, $\alpha = 0.05$, $n$ =30 and click [Execute] button.
- Result of testing hypothesis, critical value $C$ and $\beta$, will be shown.



# 7.4 Testing Hypothesis with $\alpha$ and $\beta$ simultaneously

[Example 7.4.2] In [Example 7.1.1], if the null hypothesis is not changed, but the alternative hypothesis is changed as follows:

$H_0$: $\mu$ = 1500, $H_1$ : $\mu$ = 1580

1) Calculate the probability of the type 2 error if the significance level is 5%.

2) Check this result using 『eStatU』.

〈Answer〉

1) Although the alternative hypothesis has been changed to $H_1$ : $\mu$ = 1580, decision rule will not be changed because $H_1$ is the ype as $H_1$ : $\mu$ 〉 1580.

‘If $\overline{X}$ 〈 1500 + (1.645) $\sqrt{\frac{200^2}{30}}$ = 1560.06, reject $H_0$’

- Hence, the probability of type 2 error is as follows:

$\beta$ = P( $\overline{X}$ 〈 1560.06 | $H_1$ is true)

= P( ($\overline{X}$ −1580) / $\sqrt{\frac{200^2}{30}}$ 〈 (1560.06−1580) / $\sqrt{\frac{200^2}{30}}$ )

= P( Z 〈 −0.546) = 0.293

# 7.4 Testing Hypothesis with $\alpha$ and $\beta$ simultaneously

## 7.4.1 $\beta$ and the power of test

- Discriminating ability of two hypothesis is compared by using the power of a test.

  Power = 1 − (Probability of the type 2 error) = 1 − $\beta$

  Large power increases discriminating ability of the test.

- Power of a test can be obtained for any $\mu_1$ of $H_1 : \mu = \mu_1$. It means that the power is a function over the value of and it is called a power function.

- A function of the probability that the null hypothesis is correct when the null hypothesis is true is called an operating characteristic function.

  Operating characteristic function = 1 − $\alpha$

[Example 7.4.3] In [Example 7.1.1], calculate the power of the following alternative hypothesis. Use $\alpha$ = 0.05. By using this, approximate the power function.

1) $H_1$ : $\mu$ = 1500  2) $H_1$ : $\mu$ = 1510  3) $H_1$ : $\mu$ = 1520
4) $H_1$ : $\mu$ = 1530  5) $H_1$ : $\mu$ = 1540  6) $H_1$ : $\mu$ = 1550
7) $H_1$ : $\mu$ = 1560  8) $H_1$ : $\mu$ = 1570  9) $H_1$ : $\mu$ = 1580
10) $H_1$ : $\mu$ = 1590  11) $H_1$ : $\mu$ = 1600  12) $H_1$ : $\mu$ = 1610

〈Answer〉

1) Although the alternative hypothesis are different, decision rule will not be changed because $H_1$ is the ype as $H_1$ : $\mu$ 〉 1580.

'If $\overline{X}$ 〈 1500 + (1.645) $\sqrt{\frac{200^2}{30}}$ = 1560.06, reject $H_0$'

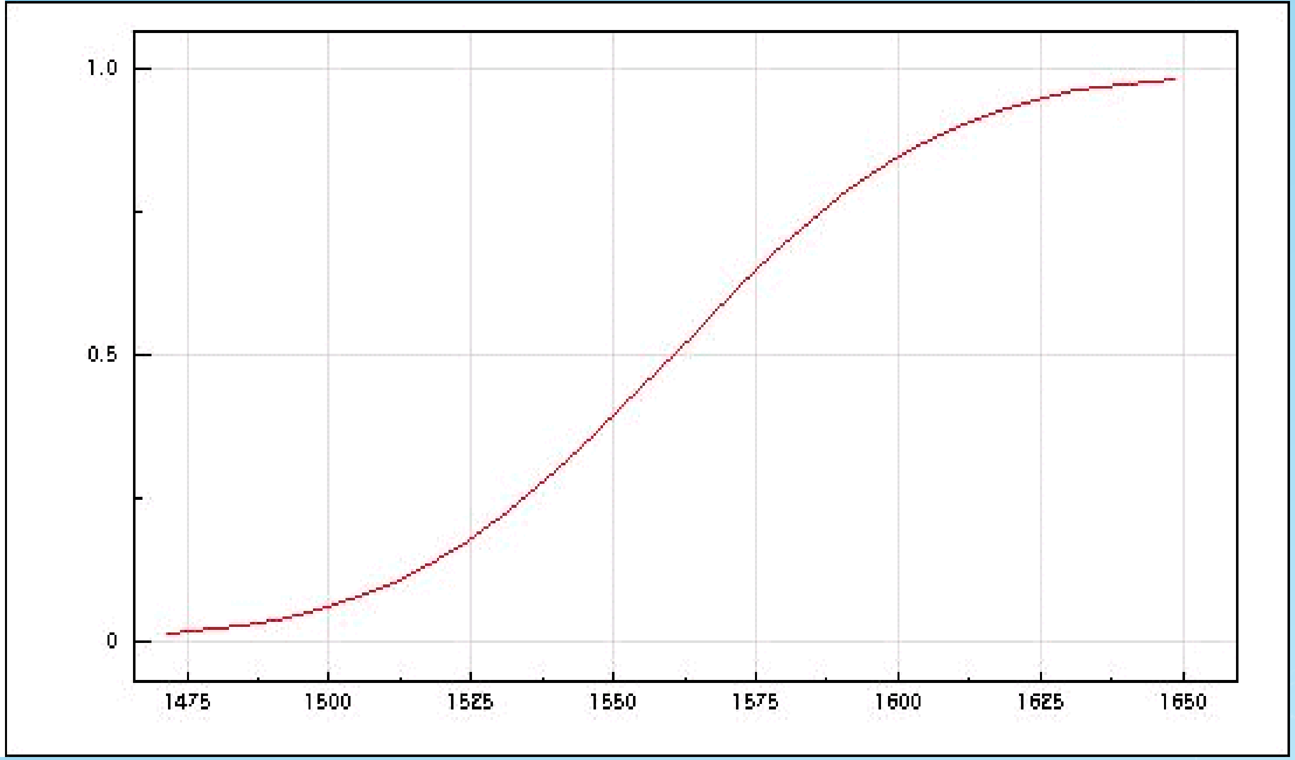
- Hence, if we calculate the probability of the type 2 error as [Example 7.4.2], the power of each test is as follows:

# 7.4 Testing Hypothesis with $\alpha$ and $\beta$ simultaneously

## 〈Answer of Ex 7.4.3〉

| Alternative Hypothesis | $\beta$ | Power = 1 - $\beta$ |
|---|---|---|
| 1) $H_1 : \mu = 1500$ | 0.95 | 0.05 |
| 2) $H_1 : \mu = 1510$ | 0.91 | 0.09 |
| 3) $H_1 : \mu = 1520$ | 0.86 | 0.14 |
| 4) $H_1 : \mu = 1530$ | 0.79 | 0.21 |
| 5) $H_1 : \mu = 1540$ | 0.71 | 0.29 |
| 6) $H_1 : \mu = 1550$ | 0.61 | 0.39 |
| 7) $H_1 : \mu = 1560$ | 0.50 | 0.50 |
| 8) $H_1 : \mu = 1570$ | 0.39 | 0.61 |
| 9) $H_1 : \mu = 1580$ | 0.29 | 0.71 |
| 10) $H_1 : \mu = 1590$ | 0.21 | 0.79 |
| 11) $H_1 : \mu = 1600$ | 0.14 | 0.86 |
| 12) $H_1 : \mu = 1610$ | 0.09 | 0.91 |

- **Power function**



# 7.4 Testing Hypothesis with $\alpha$ and $\beta$ simultaneously

## 7.4.2 Testing Hypothesis with $\alpha$ and $\beta$

[Example 7.4.4] Consider the testing hypothesis on the bulb life such as $H_0$: $\mu$ = 1500, $H_1$ : $\mu$ = 1570. Find sample size $n$ and decision rule which satisfies $\alpha$ = 5% and $\beta$ = 10%. Assume $\sigma$ = 200 hours.

〈Answer〉

- Let $n$ be the sample size and $C$ be the critical value of a decision rule.
- Probability of the type 1 error $\alpha$ and type 2 error $\beta$ are defined as follows:

$$\alpha = P(\bar{X} > C \mid H_0 \text{ is true})$$
$$\beta = P(\bar{X} < C \mid H_1 \text{ is true})$$

# 7.4 Testing Hypothesis with $\alpha$ and $\beta$ simultaneously

⟨Answer of Example 7.4.4⟩

- If $H_0$ is true, $\bar{X} \sim N(1500, \frac{200^2}{n})$ and if $H_1$ is true, $\bar{X} \sim N(1570, \frac{200^2}{n})$ . If $\alpha$ = 0.05 and $\beta$ = 0.10, then $z_{0.05}$ = 1.645 and $z_{0.10}$ = −1.280. Hence $n$ and $C$ should satisfy both of the following equations.

$$C = 1500 + 1.645 \times \frac{200}{\sqrt{n}}$$
$$C = 1570 - 1.280 \times \frac{200}{\sqrt{n}}$$

- By solving two system of equations, solution is $n$ = 69.8, $C$ = 1539.4. i.e., the sample size is 70 approximately and the decision rule is as follows:

‘If $\bar{X}$ 〉 1539.4, then reject $H_0$’

# 7.4 Testing Hypothesis with $\alpha$ and $\beta$ simultaneously

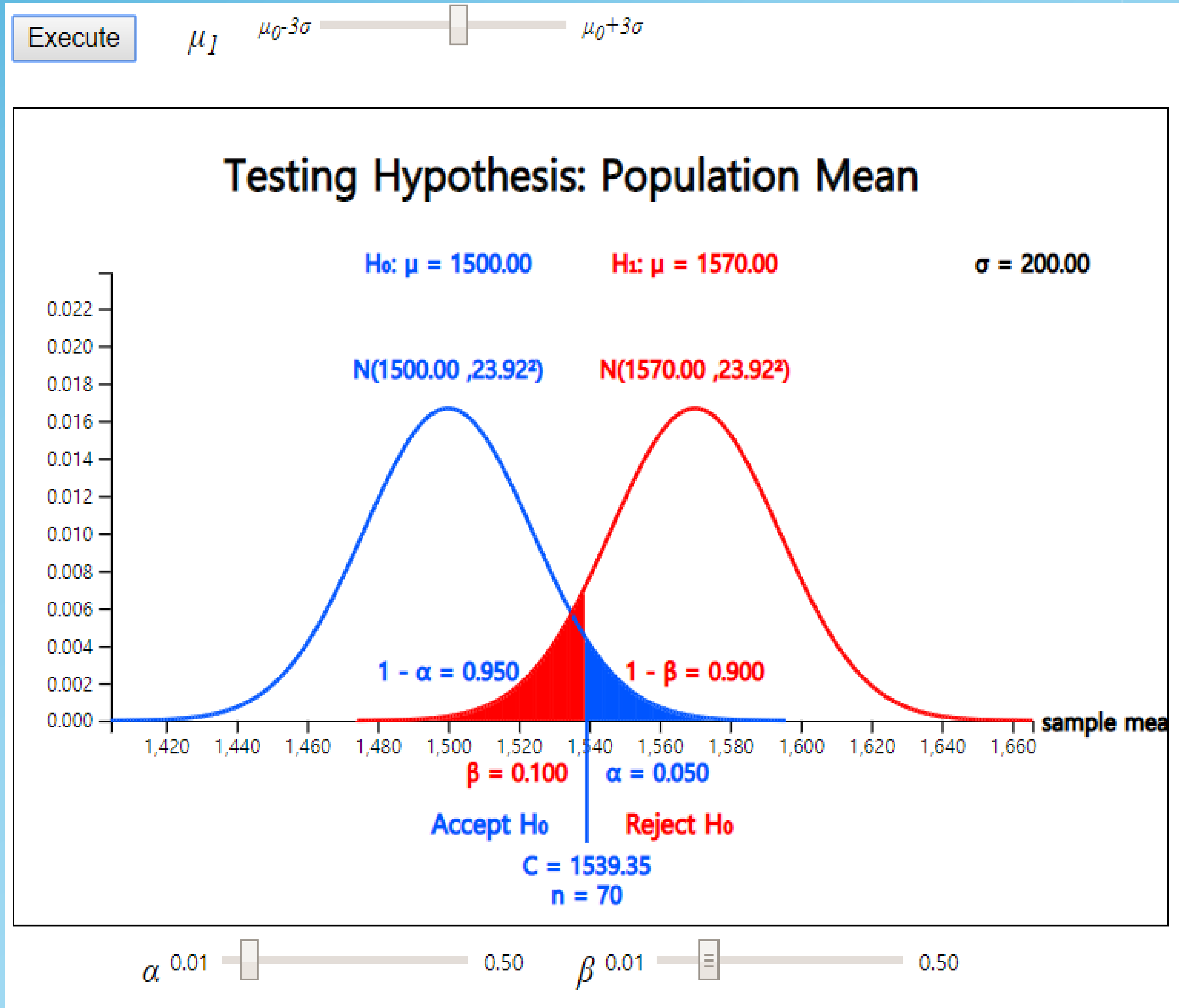
## ⟨Answer of Ex 7.4.4⟩

**Testing μ - C, n** Menu

[Hypothesis] $H_o : \mu = \mu_o$ 1500 $H_1 : \mu = \mu_1$ 1570

Population Standard Deviation $\sigma =$ 200

Type 1 Error $\alpha =$ 0.050 Type 2 Error $\beta =$ 0.100



# Thank you