Introduction to Statistics and Data Science using *eStat*

Chapter 9 Testing Hypothesis for Several Population Means

9.2 Design of Experiments for Sampling

9.2.1 Completely Randomized Design
9.2.2 Randomized Block Design
9.2.3 Latin Square Design

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9.1 Analysis of Variance for Experiments of Single Factor

9.1.1 Multiple Comparison

9.1.2 Residual Analysis

9.2 Design of Experiments for Sampling

9.2.1 Completely Randomized Design

9.2.2 Randomized block design

9.3 Analysis of Variance for Experiments of Two Factors

9.2 Design of Experiments for Sampling

9.2.1 Completely Randomized Design

- Design of experiments to have little impact from other factors.
- One way to do this is to make the whole experiments random.
- Example: Compare gas milage of three cars (A, B, C) with 5 drivers

Driver	1	2	3	4	5
Car Type	B B C	A C B	B A A	C A B	A C C

9.2.2 Randomized Block Design

Driver	1	2	3	4	5
Car Type (gas mileage)	A(22.4) C(20.2) B(16.3)	B(12.6) C(15.2) A(16.1)	C(18.7) A(19.7) B(15.9)	A(21.1) B(17.8) C(18.9)	A(24.5) C(23.8) B(21.0)

9.2.2 Randomized Block Design

- Statistical model of the randomized block design:

$$Y_{ij} = \mu + \alpha_i + B_j + \epsilon_{ij}, \qquad i = 1, 2, \dots, k, j = 1, 2, \dots, b$$

B_j : effect of j^{th} level of the block variable

- In the randomized block design, the total variation is divided into as follows:

$$Y_{ij} - \bar{Y}_{..} = (Y_{ij} - \bar{Y}_{i.} - \bar{Y}_{.j} + \bar{Y}_{..}) + (\bar{Y}_{i.} - \bar{Y}_{..}) + (\bar{Y}_{.j} - \bar{Y}_{..})$$

9.2 Design of Experiments for Sampling

9.2.2 Randomized Block Design

Division of sum of squares and degree of freedom

Sum of squares :	SST	=	SSE	+	SSTr	+	SSB
Degree of freedom :	$bk-1$	=	$(b-1)(k-1)$	+	$(k-1)$	+	$(b-1)$

Table 9.2.3 Analysis of Variance Table of the randomized block design

Variation	Sum of Squares	Degree of freedom	Mean Squares	F value
Treatment	SSTr	$k-1$	$\mathrm{MSTr} = \dfrac{\mathrm{SSTr}}{k-1}$	$F_0 = \dfrac{MSTr}{MSE}$
Block	SSB	$b-1$	$\mathrm{MSB} = \dfrac{\mathrm{SSB}}{b-1}$	
Error	SSE	$(b-1)(k-1)$	$\mathrm{MSE} = \dfrac{\mathrm{SSE}}{(b-1)(k-1)}$	
Total	SST	$bk-1$		

Total sum of squares, degree of freedom $bk-1$

$$\mathrm{SST} = \sum_{i=1}^{k}\sum_{j=1}^{b}(Y_{ij} - \overline{Y}_{..})^2$$

Error sum of squares, degree of freedom $(b-1)(k-1)$

$$\mathrm{SSE} = \sum_{i=1}^{k}\sum_{j=1}^{b}(Y_{ij} - \overline{Y}_{i.} - \overline{Y}_{.j} + \overline{Y}_{..})^2$$

Treatment sum of squares, degree of freedom $k-1$

$$\mathrm{SSTr} = \sum_{i=1}^{k}\sum_{j=1}^{b}(\overline{Y}_{i.} - \overline{Y}_{..})^2 = b\sum_{i=1}^{k}(\overline{Y}_{i.} - \overline{Y}_{..})^2$$

Block sum of squares, degree of freedom $b-1$

$$\mathrm{SSB} = \sum_{i=1}^{k}\sum_{j=1}^{b}(\overline{Y}_{.j} - \overline{Y}_{..})^2 = k\sum_{j=1}^{b}(\overline{Y}_{.j} - \overline{Y}_{..})^2$$

9.2 Design of Experiments for Sampling

[Example 9.2.1] Table 9.2.4 is the rearrangement of the fuel mileage data in Table 9.2.2 measured by five drivers and car types.

Driver		1	2	3	4	5	Average
Car Type	A	22.4	16.1	19.7	21.1	24.5	20.76
	B	16.3	12.6	15.9	17.8	21.0	16.72
	C	20.2	15.2	18.7	18.9	23.8	19.36
Average		19.63	14.63	18.10	19.27	23.10	18.947

1) Assuming that this data has been measured by the completely design, use 『eStat』 to do the analysis of variance whether the three car types have the same fuel mileage.

2) Assuming that this data has been measured by the randomized block design, use 『eStat』 to do the analysis of variance whether the three car types have the same fuel mileage.

9.2 Design of Experiments for Sampling

<Answer of [Example 9.2.1]>

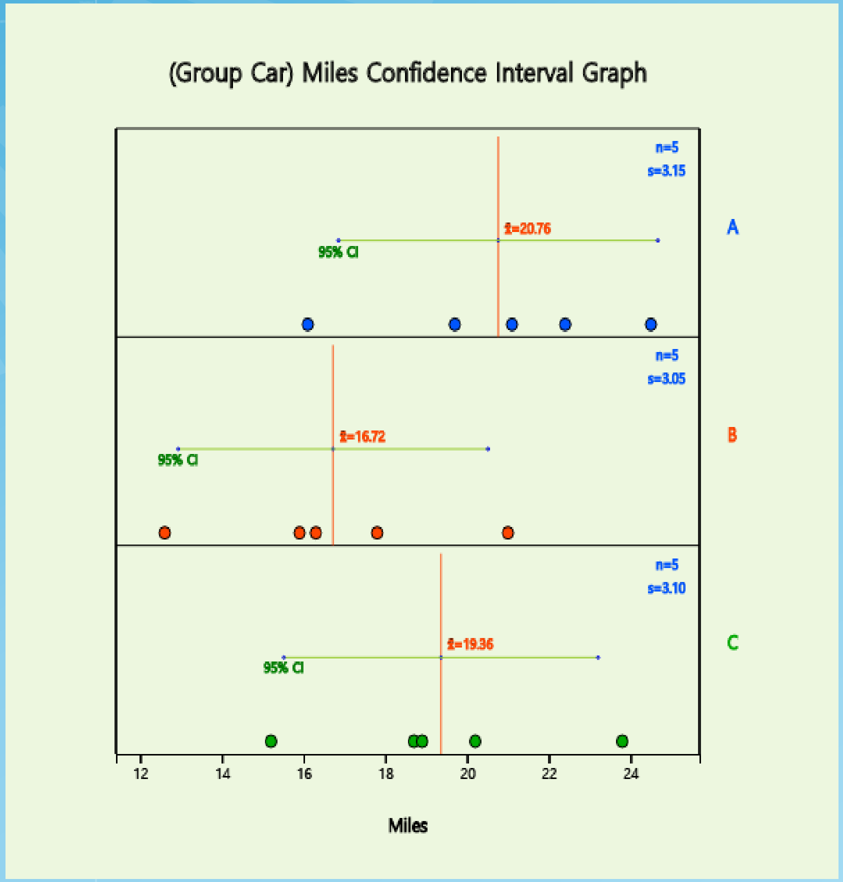
File: Ex921GasMileage.csv

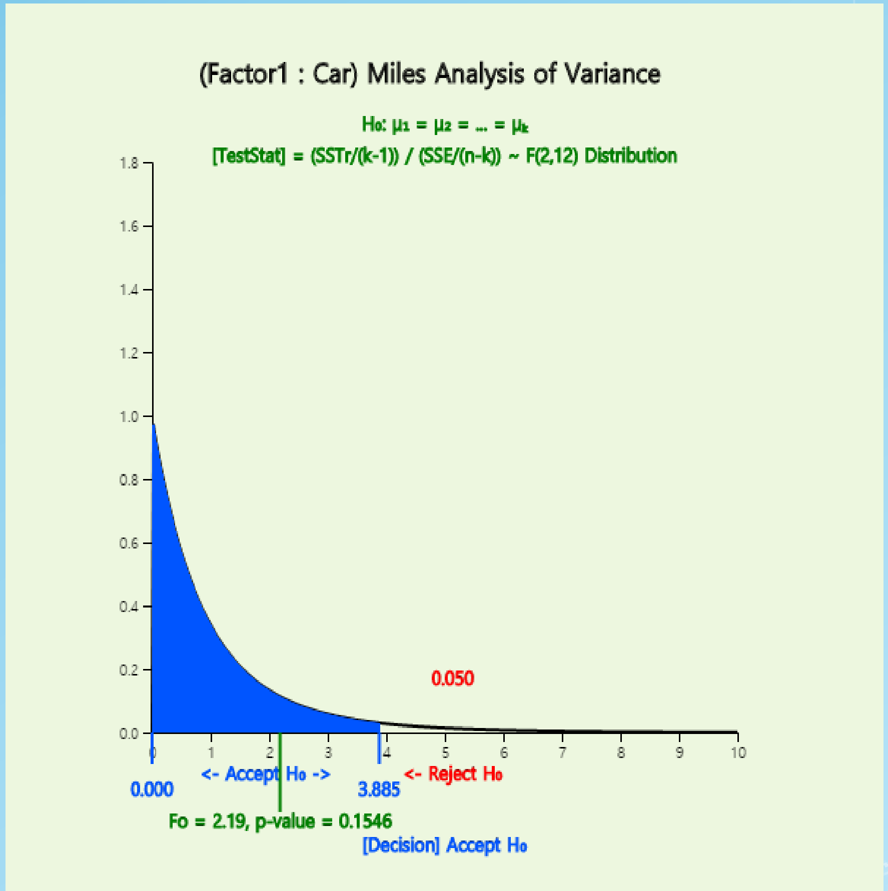
Analysis Var: 3: Miles — by Group: 1: Car

(Selected data: Raw Data) (Select up to two g

SelectedVar: V3 by V1,

	Car	Driver	Miles	V4
1	A	1	22.4	
2	A	2	16.1	
3	A	3	19.7	
4	A	4	21.1	
5	A	5	24.5	
6	B	1	16.3	
7	B	2	12.6	
8	B	3	15.9	
9	B	4	17.8	
10	B	5	21.0	
11	C	1	20.2	
12	C	2	15.2	
13	C	3	18.7	
14	C	4	18.9	
15	C	5	23.8	





9.2 Design of Experiments for Sampling

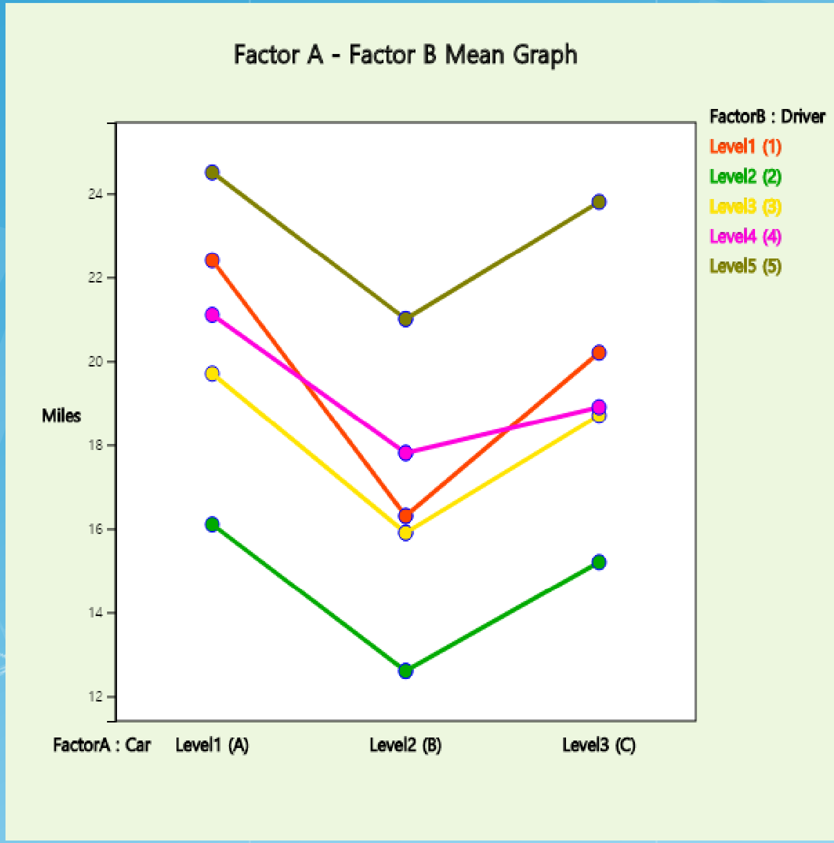
<Answer of [Example 9.2.1]>

Analysis of Variance					
Factor	Sum of Squares	deg of freedom	Mean Squares	F value	p value
Treatment	42.085	2	21.043	2.190	0.1546
Error	115.312	12	9.609		
Total	157.397	14			

Multiple Comparison	Analysis Var	(Miles)	Group Name	(Car)
Mean Difference (95%HSD)	1 (A) 20.76	2 (B) 16.72	3 (C) 19.36	
1 (A) 20.76		4.04 (5.23)	1.40 (5.23)	
2 (B) 16.72	4.04 (5.23)		2.64 (5.23)	
3 (C) 19.36	1.40 (5.23)	2.64 (5.23)		

9.2 Design of Experiments for Sampling

<Answer of [Example 9.2.1]>



Two-dimension Statistics

Observation Mean Std Dev	Factor B (Driver) Level1 (1)	Factor B (Driver) Level2 (2)	Factor B (Driver) Level3 (3)	Factor B (Driver) Level4 (4)	Factor B (Driver) Level5 (5)	Factor A Level i Total
FactorA (Car) Level1 (A)	1 22.400 NaN	1 16.100 NaN	1 19.700 NaN	1 21.100 NaN	1 24.500 NaN	5 20.760 3.148
FactorA (Car) Level2 (B)	1 16.300 NaN	1 12.600 NaN	1 15.900 NaN	1 17.800 NaN	1 21.000 NaN	5 16.720 3.054
FactorA (Car) Level3 (C)	1 20.200 NaN	1 15.200 NaN	1 18.700 NaN	1 18.900 NaN	1 23.800 NaN	5 19.360 3.097
Factor B Level j Total	3 19.633 3.089	3 14.633 1.818	3 18.100 1.970	3 19.267 1.680	3 23.100 1.852	15 18.947 3.353
Missing Observations	0					

Analysis of Variance

Factor	Sum of Squares	deg of freedom	Mean Squares	F value	p value
Factor A (Car)	42.085	2	21.043	43.447	< 0.0001
Factor B (Driver)	111.437	4	27.859	57.521	< 0.0001
Error	3.875	8	0.484		
Total	157.397	14			

9.2 Design of Experiments for Sampling

9.2.3 Latin Square Design

- **Two block variables:**

Table 9.2.6 Fuel mileage data by four drivers and four road types of four car types (A, B, C, D)

		Column 1 Road 1	Column 2 Road 2	Column 3 Road 3	Column 4 Road 4
Row 1	Driver 1	A	B	C	D
Row 2	Driver 2	B	C	D	A
Row 3	Driver 3	C	D	A	B
Row 4	Driver 4	D	A	B	C

Null hypothesis $H_0 : \mu_1 = \mu_2 = \cdots = \mu_r$

Alternative hypothesis H_1 : At least one pair of μ_i is not equal

9.2 Design of Experiments for Sampling

9.2.3 Latin Square Design

- Statistical model of the randomized block design:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \epsilon_{ijk}, \qquad i, j, k = 1, 2, \dots, r$$

α_i : effect of i^{th} level of row block variable

β_j : effect of j^{th} level of column block variable

γ_k : effect of k^{th} treatment variable

- In the Latin square design, total variation is divided as follows:

$$Y_{ijk} - \bar{Y}_{...} = (Y_{ijk} - \bar{Y}_{i..} - \bar{Y}_{.j.} - \bar{Y}_{..k} + 2\bar{Y}_{...}) + (\bar{Y}_{i..} - \bar{Y}_{...}) + (\bar{Y}_{.j.} - \bar{Y}_{...}) + (\bar{Y}_{..k} - \bar{Y}_{...})$$

9.2 Design of Experiments for Sampling

9.2.3 Latin Square Design

- **Statistical model of the randomized block design:**

$$Y_{ij} = \mu + \alpha_i + B_j + \epsilon_{ij}, \qquad i = 1, 2, \dots, k, j = 1, 2, \dots, b$$

B_j : effect of j^{th} level of the block variable

- **In the randomized block design, the total variation is divided into as follows:**

$$Y_{ij} - \bar{Y}_{..} = (Y_{ij} - \bar{Y}_{i.} - \bar{Y}_{.j} + \bar{Y}_{..}) + (\bar{Y}_{i.} - \bar{Y}_{..}) + (\bar{Y}_{.j} - \bar{Y}_{..})$$

9.2 Design of Experiments for Sampling

9.2.3 Latin Square Design

Total sum of squares, degrees of freedom r^3-1

$$SST = \sum_{i=1}^{r}\sum_{i=1}^{r}\sum_{j=1}^{r}(Y_{ijk}-\overline{Y}_{...})^2$$

Error sum of squares, degrees of freedom r^2-3r+2

$$SSE = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}(Y_{ijk}-\overline{Y}_{i..}-\overline{Y}_{.j.}-\overline{Y}_{..k}+2\overline{Y}_{...})^2$$

Row sum of squares, degrees of freedom $r-1$

$$SSR = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}(\overline{Y}_{i..}-\overline{Y}_{...})^2$$

Column sum of squares, degrees of freedom $r-1$

$$SSC = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}(\overline{Y}_{.j.}-\overline{Y}_{...})^2$$

Treatment sum of squares, degrees of freedom $r-1$

$$SSTr = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}(\overline{Y}_{..k}-\overline{Y}_{...})^2$$

Table 9.2.8 ANOVA table of the Latin square design

Variation	Sum of Squares	Degrees of freedom	Mean Squares	F value
Treatment	SSTr	$r-1$	$MSTr = \frac{SSTr}{r-1}$	$F_0 = \frac{MSTr}{MSE}$
Row	SSR	$r-1$	$MSR = \frac{SSR}{r-1}$	
Column	SSC	$r-1$	$MSC = \frac{SSC}{r-1}$	
Error	SSE	r^2-3r+2	$MSE = \frac{SSE}{r^2-3r+2}$	
Total	SST	r^3-1		

9.2 Design of Experiments for Sampling

9.2.3 Latin Square Design

Example 9.2.2

Table 9.2.9 is the fuel mileage data of four car types (A, B, C, D) measured by four drivers and four road types with Latin square design.

Table 9.2.9 Fuel mileage data by four drivers and four road types of four car types (A, B, C, D)

		Column 1 Road 1	Column 2 Road 2	Column 3 Road 3	Column 4 Road 4
Row 1	Driver 1	A(22)	B(16)	C(19)	D(21)
Row 2	Driver 2	B(24)	C(16)	D(12)	A(15)
Row 3	Driver 3	C(17)	D(21)	A(20)	B(15)
Row 4	Driver 4	D(18)	A(18)	B(23)	C(22)

Use 「eStatU」 to do the analysis of variance whether the four car types have the same fuel mileage.

9.2 Design of Experiments for Sampling

9.2.3 Latin Square Design

Testing Hypothesis ANOVA - Latin Square Design | Menu

[Hypothesis] $H_o: \mu_A = \mu_B = \ldots = \mu_r$

H_1 : *At least one pair of means is different*

[Test Type] *F test* (ANOVA)

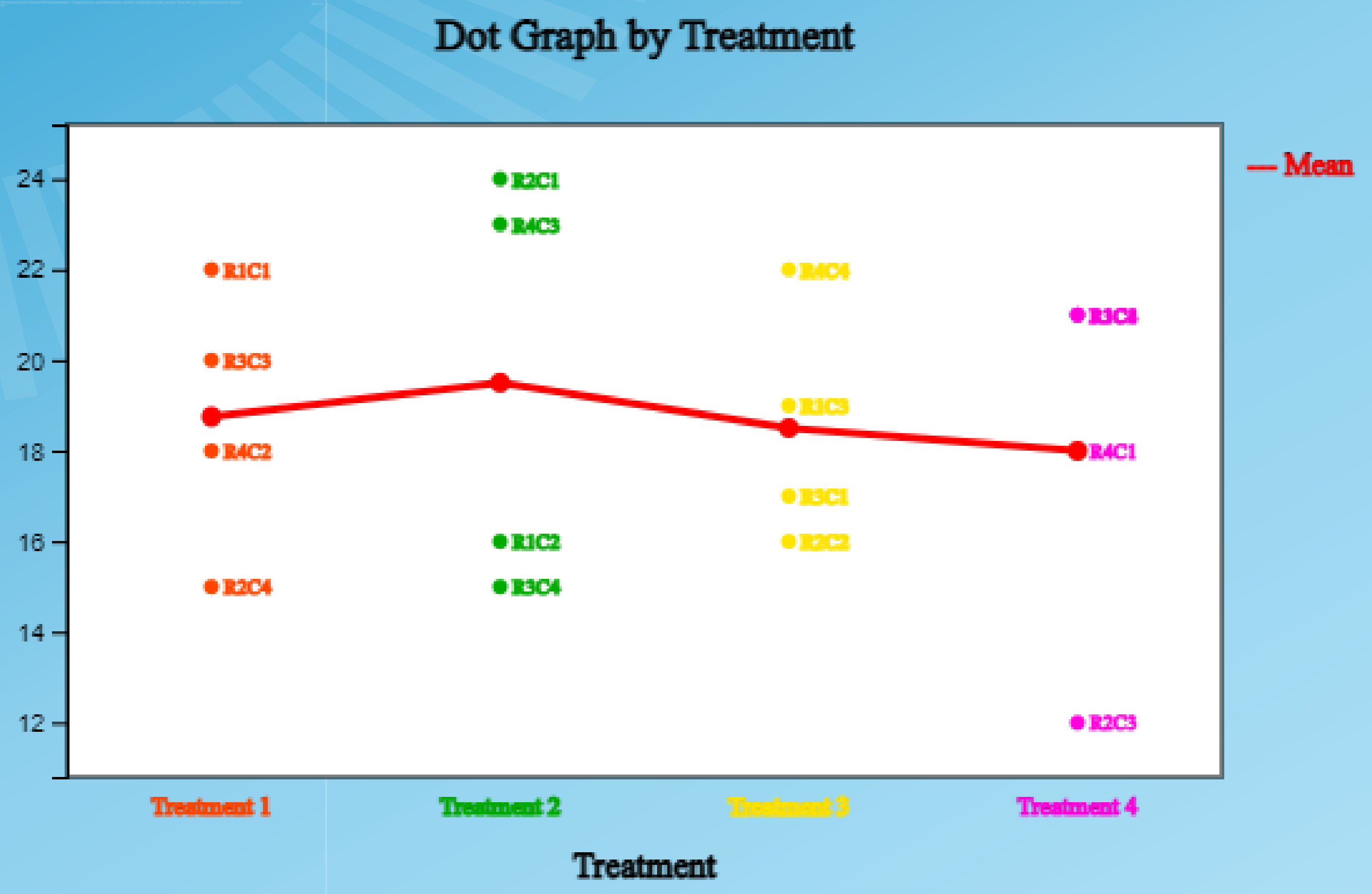
[Sample Data] Treatment r = 4 (*A,B,C,D*)

	Column1		Column2		Column3		Column4		Column5		Column6		Row
Row1	A	22	B	16	C	19	D	21				$\bar{x}_{1..}$	19.500
Row2	B	24	C	16	D	12	A	15				$\bar{x}_{2..}$	16.750
Row3	C	17	D	21	A	20	B	15				$\bar{x}_{3..}$	18.250
Row4	D	18	A	18	B	23	C	22				$\bar{x}_{4..}$	20.250
Row5												$\bar{x}_{5..}$	
Row6												$\bar{x}_{6..}$	
Column	$\bar{x}_{.1.}$	20.250	$\bar{x}_{.2.}$	17.750	$\bar{x}_{.3.}$	18.500	$\bar{x}_{.4.}$	18.250	$\bar{x}_{.5.}$		$\bar{x}_{.6.}$		
Treatment	$\bar{x}_{..1}$	18.750	$\bar{x}_{..2}$	19.500	$\bar{x}_{..3}$	18.500	$\bar{x}_{..4}$	18.000	$\bar{x}_{..5}$		$\bar{x}_{..6}$		$\bar{x}_{...}$	18.688

Execute

9.2 Design of Experiments for Sampling

9.2.3 Latin Square Design



9.2 Design of Experiments for Sampling

9.2.3 Latin Square Design

Analysis of Variance					
Factor	Sum of Squares	deg of freedom	Mean Squares	F value	p value
Treatment	4.688	3	1.563	0.075	0.9710
Row Var	28.188	3	9.396		
Col Var	14.188	3	4.729		
Error	124.375	6	20.729		
Total	171.438	15			

Thank you