Introduction to Statistics and Data Science using *eStat*

Chapter 10 Nonparametric Testing Hypothesis

10.1 Nonparametric Test for Location of Single Population

10.1.2 Wilcoxon Signed Rank Sum Test

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10.1 Nonparametric Test for Location Parameter of Single Population

10.1.2 Wilcoxon Signed Rank Sum Test

[Example 10.1.2] A bag of cookies is marked with a weight of 200g. Ten bags are randomly selected from several retailers and examined their weights.

203 204 197 195 201 205 198 199 194 207

- Can you say that there are as many cookies in the bag as weight marked?
- Test the hypothesis by using the Wilcoxon Signed Rank Sum Test with the significance level of 5%.
- Check the result of the above test using 『eStatU』.

10.1 Nonparametric Test for Location Parameter of Single Population

[Answer of Example 10.1.2]

- Hypothesis $H_0 : M = 200,\ H_1 : M \neq 200$

Sample data	203	204	197	195	201	205	198	199	194	207
Sign data	+	+	-	-	+	+	-	-	-	+
|data – 200|	3	4	3	5	1	5	2	1	6	7
Rank of |data – 200|	4.5	6	4.5	7.5	1.5	7.5	3	1.5	9	10
Rank sum of '+' sign	$R_+ = 4.5 + 6 + 1.5 + 7.5 + 10 = 29.5$									

10.1 Nonparametric Test for Location Parameter of Single Population

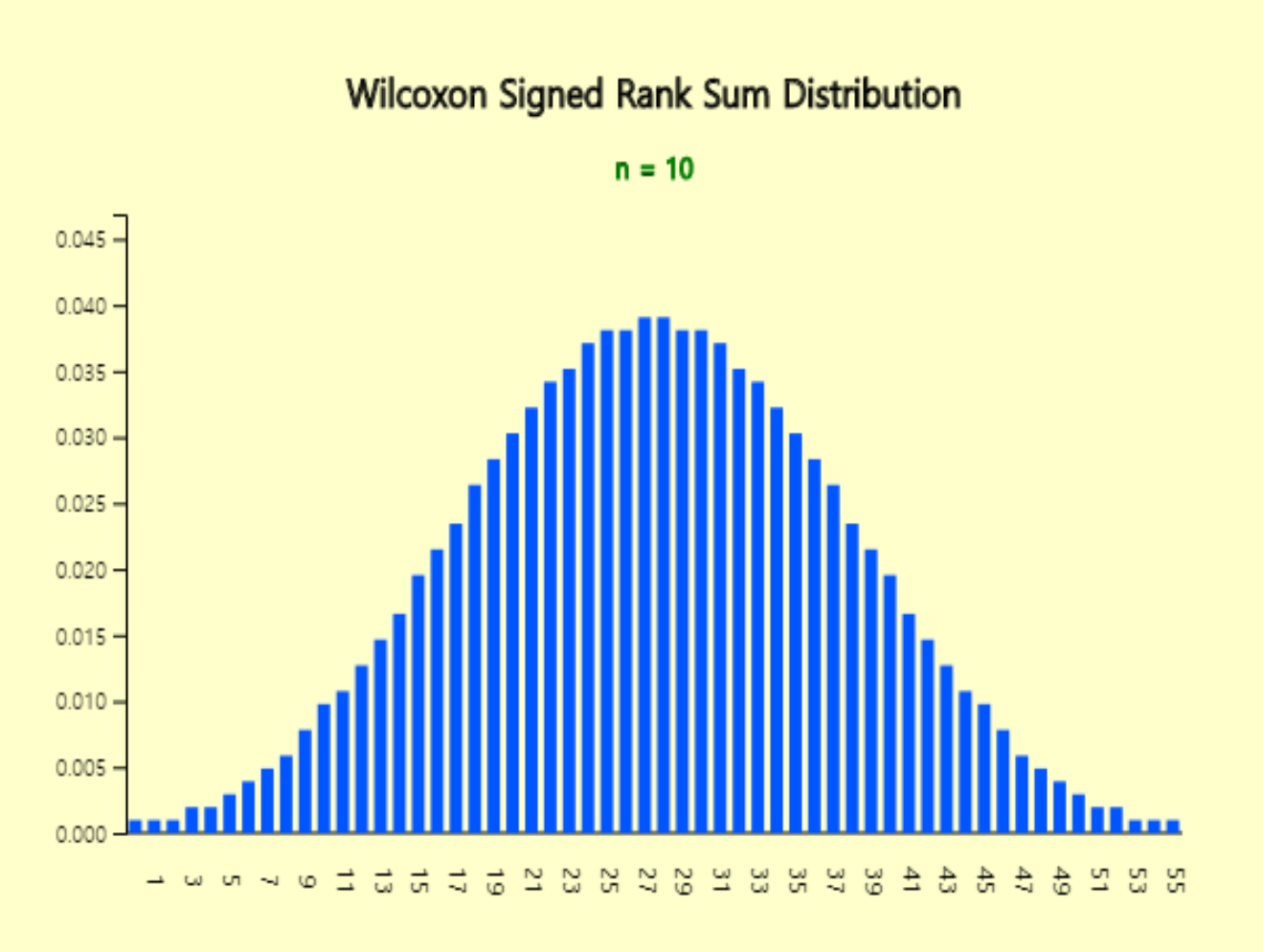
[Answer of Example 10.1.2]

- **All possible cases of R_+ = 'rank sum of + sign data'**

Number of data with + sign	All possible combination of ranks	All possible rank sum of R_+
0	0	0
1	{1}, {2}, … , {10}	1, 2, … , 10
2	{1,2}, {1,3}, … , {1,10}, {2,3}, … , {2,10}, … {9,10}	3, 4, … , 11, 5, … , 12, … 19
…	…	…
10	{1,2, ,10}	55

10.1 Nonparametric Test for Location Parameter of Single Population

- **Distribution of Wilcoxon signed rank sum when n = 10**



Wilcoxon Signed Rank Sum Distribution			n = 10
x	P(X = x)	P(X x)	P(X x)
0	0.0010	0.0010	1.0000
1	0.0010	0.0020	0.9990
2	0.0010	0.0029	0.9980
3	0.0020	0.0049	0.9971
4	0.0020	0.0068	0.9951
5	0.0029	0.0098	0.9932
6	0.0039	0.0137	0.9902
7	0.0049	0.0186	0.9863
8	0.0059	0.0244	0.9814
9	0.0078	0.0322	0.9756
47	0.0059	0.9814	0.0244
48	0.0049	0.9863	0.0186
49	0.0039	0.9902	0.0137
50	0.0029	0.9932	0.0098
51	0.0020	0.9951	0.0068
52	0.0020	0.9971	0.0049
53	0.0010	0.9980	0.0029
54	0.0010	0.9990	0.0020
55	0.0010	1.0000	0.0010

10.1 Nonparametric Test for Location Parameter of Single Population

- Since P(X ≤ 8) = 0.0244, P(X ≥ 47) = 0.0244, the decision rule is:

  'If $R_+ \leq 8.5$ or $R_+ \geq 46.5$, then reject H_0 '

- Since $R_+ = 29.5$ in this problem, we can not reject H_0 .

Signed Rank Sum Test Menu

[Hypothesis] $H_0 : M = M_0$ 200

$H_1 : M \neq M_0$ $H_1 : M > M_0$ $H_1 : M < M_0$

[Test Type] Signed Rank Sum Test

Significance Level α = 5% 1%

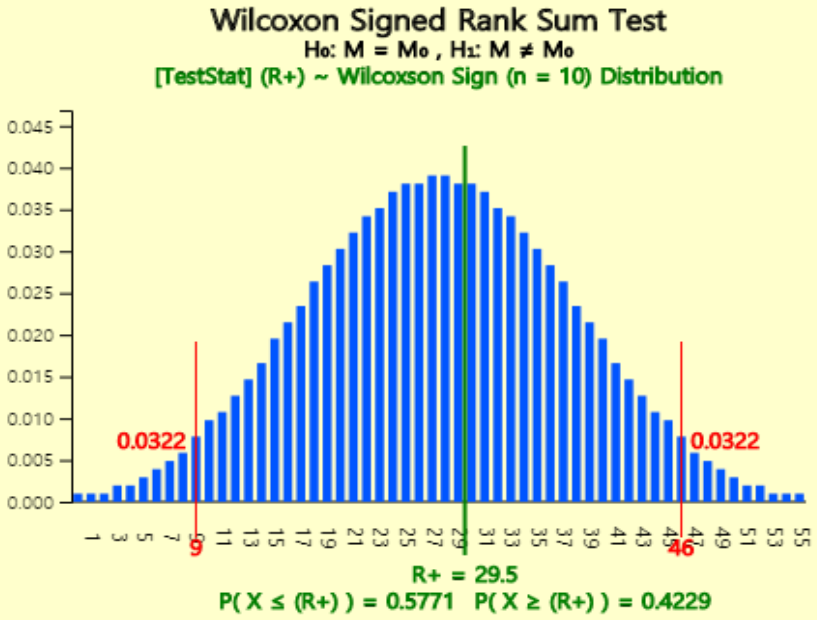
[Sample Data]

Sample 203 204 197 195 201 205 198 199 194 207

[Sample Statistics]

Sample Size	n_- =	5	n_+ =	5
Rank Sum	R_- =	25.5	R_+ =	29.5

Execute *If n ≤ 20 Wilcoxon Rank Sum Test, n > 20 Nomal Approximation Test*



10.1 Nonparametric Test for Location Parameter of Single Population

File EX100101_CookieWeight.csv

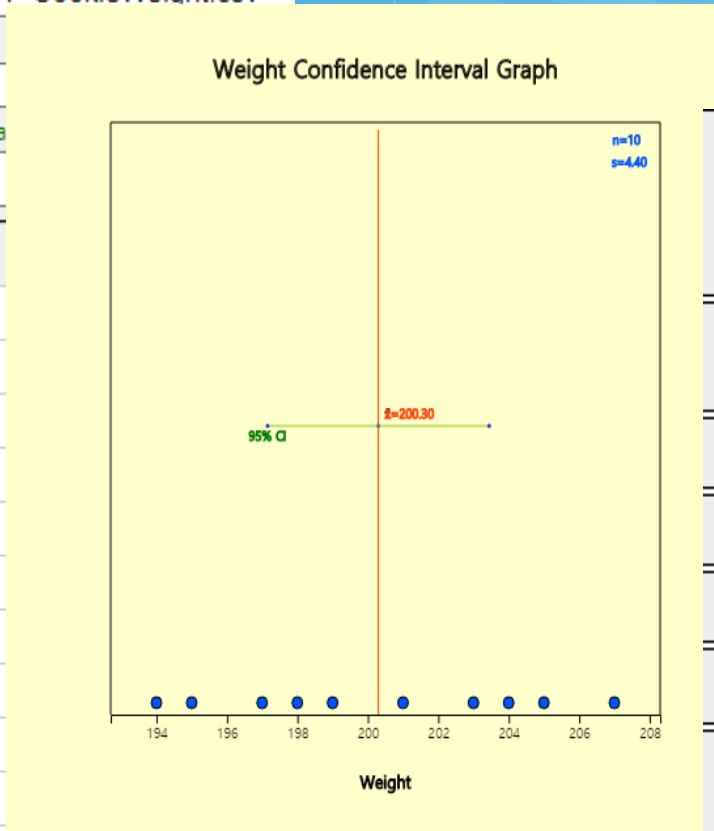
Analysis Var

1: Weight

(Selected data: Raw Data

SelectedVar V1

	Weight	V2
1	203	
2	204	
3	197	
4	195	
5	201	
6	205	
7	198	
8	199	
9	194	
10	207	



Wilcoxon Signed Rank Sum Test	Analysis Var	Weight			
Statistics	Total Observation	Test Observation	-Group Observation	+Group Observation	
	10	10	5	5	
Missing Observations	0				
Hypothesis					
$H_0 : M = M_0$	μ_0	[TestStat]	Rank Sum R+	$P(X \le R+)$	$P(X \ge R+)$
$H_1 : M \ne M_0$	200.00	+Group Rank Sum(R+)	29.50	0.5771	0.4229

10.1 Nonparametric Test for Location Parameter of Single Population

Table 10.1.5 Decision rule of Wilcoxon signed rank sum test

Type of Hypothesis	Decision Rule Test Statistic R_+ = Rank sum of + sign data of $\lvert x_i - M_0 \rvert$
1) $H_0 : M = M_0$ $H_1 : M > M_0$	If $R_+ > w_+(n)_\alpha$, then reject H_0, else accept H_0
2) $H_0 : M = M_0$ $H_1 : M < M_0$	If $R_+ < w_+(n)_{1-\alpha}$, then reject H_0, else accept H_0
3) $H_0 : M = M_0$ $H_1 : M \neq M_0$	If $R_+ < w_+(n)_{1-\alpha/2}$ or $R_+ > w_+(n)_{\alpha/2}$, then reject H_0, else accept H_0

- $w_+(n)$: Distribution of + rank sum of $|x_i - M_0|$
- If any of the observed values has the same value as M_0 , they are not used in test.

10.1 Nonparametric Test for Location Parameter of Single Population

Table 10.1.6 Decision rule of Wilcoxon signed rank sum test (large sample case)

Type of Hypothesis	Decision Rule Test Statistic: R_+ = Rank sum of + sign data of $\lvert x_i - M_0 \rvert$
1) $H_0 : M = M_0$ $H_1 : M > M_0$	If $\dfrac{R_+ - E(R_+)}{\sqrt{V(R_+)}} > z_\alpha$, then reject H_0, else accept H_0
2) $H_0 : M = M_0$ $H_1 : M < M_0$	If $\dfrac{R_+ - E(R_+)}{\sqrt{V(R_+)}} < -z_\alpha$, then reject H_0, else accept H_0
3) $H_0 : M = M_0$ $H_1 : M \neq M_0$	If $\left\lvert \dfrac{R_+ - E(R_+)}{\sqrt{V(R_+)}} \right\rvert > z_{\alpha/2}$, then reject H_0, else accept H_0

- $E(R_+) = \frac{n(n+1)}{4}$, $V(R_+) = \frac{n(n+1)(2n+1)}{24}$

Thank you