Introduction to Statistics and Data Science using *eStat*

Chapter 10 Nonparametric Testing Hypothesis

10.3 Nonparametric Test for Comparing Locations of Several Populations
10.3.2 Friedman Test

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10.1 Nonparametric Test for Location of Single Population
10.1.1 Sign Test
10.1.2 Wilcoxon Signed Rank Sum Test

10.2 Nonparametric Test for Comparing Locations of Two Populations
10.2.1 Independent Samples: Wilcoxon Rank Sum Test
10.2.2 Paired Samples: Wilcoxon Signed Rank Sum Test

10.3 Nonparametric Test for Comparing Locations of Several Populations
10.3.1 Completely Randomized Design: Kruskal-Wallis Test
10.3.2 Randomized block design: Friedman Test

10.3 Nonparametric Test for Location of Several Populations

10.3.2 Randomized Block Design : Friedman Test

[Example 10.3.2] The fuel mileage of the three types of cars (A, B and C) is measured using the randomized block design as following table.

1) Draw a histogram of the data to see if the fuel mileage of the three cars can be tested by a parametric method.
2) Using the Friedman test which is a nonparametric method of the randomized block design, test whether the fuel mileage of the three types of cars are different with the significance level of 5%.
3) Check the result of the above Friedman test using『eStatU』.

		Car A	Car B	Car C
Driver (Block)	1	22.4	16.3	20.2
	2	16.1	12.6	15.2
	3	19.7	15.9	18.7
	4	21.1	17.8	18.9
	5	24.5	21.0	23.8

10.3 Nonparametric Test for Location of Several Populations

<Answer of 10.3.2>

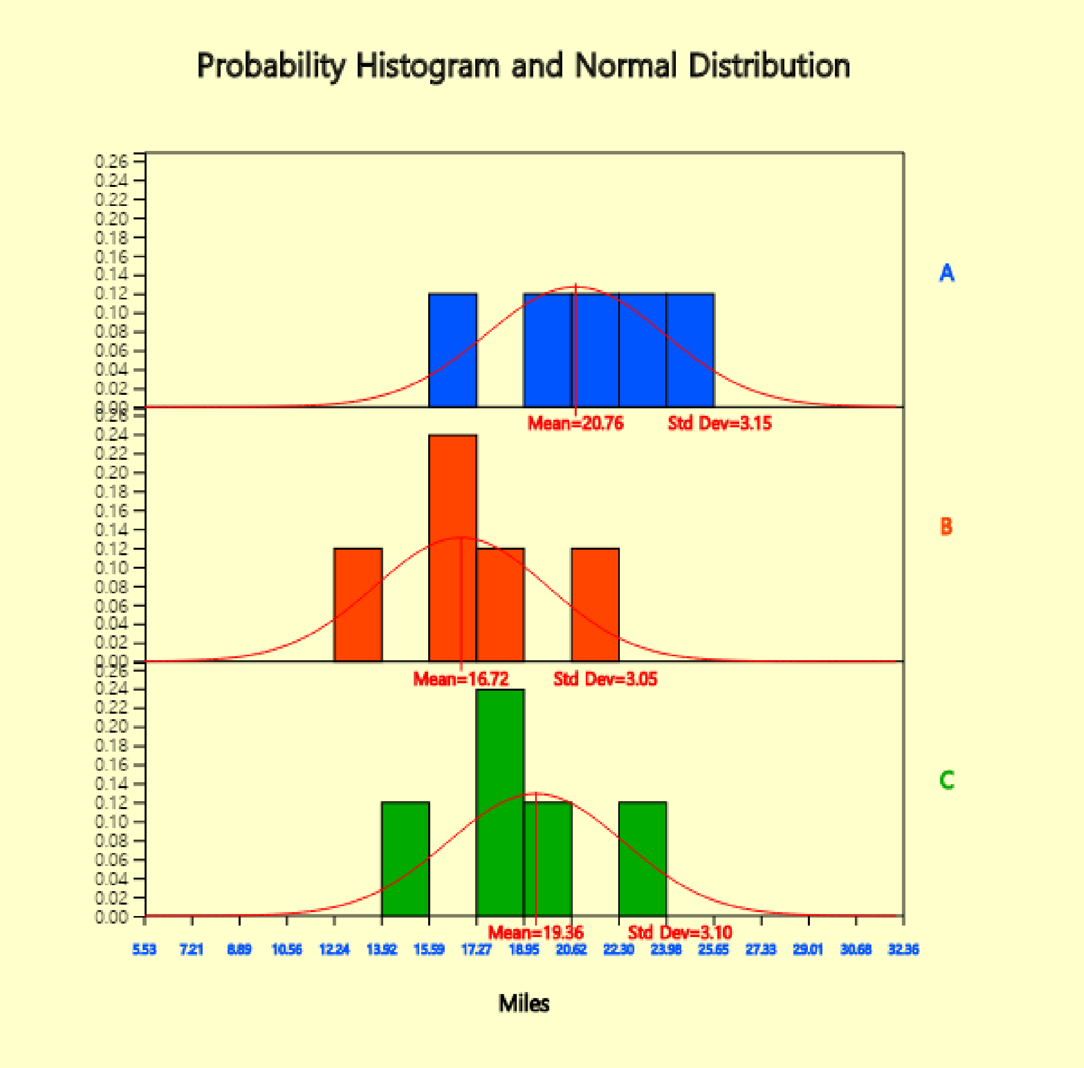
File: EX090201_GasMileage.csv

Analysis Var: 3: Miles by Group: 1: Car

(Selected data: Raw Data) (Select up to t

SelectedVar: V3 by V1,

	Car	Driver	Miles	V4
1	A	1	22.4	
2	A	2	16.1	
3	A	3	19.7	
4	A	4	21.1	
5	A	5	24.5	
6	B	1	16.3	
7	B	2	12.6	
8	B	3	15.9	
9	B	4	17.8	
10	B	5	21.0	
11	C	1	20.2	
12	C	2	15.2	
13	C	3	18.7	
14	C	4	18.9	
15	C	5	23.8	



10.3 Nonparametric Test for Location of Several Populations

<Answer of Example 10.3.2>

- Hypothesis $H_o : M_1 = M_2 = M_3$

  H_1 : At least one pair of location parameters is not the same

- Ranking in each of the block

		Car A	Car B	Car C
Driver (Block)	1	3	1	2
	2	3	1	2
	3	3	1	2
	4	3	1	2
	5	3	1	2
Sum of ranks		R_1=15	R_2=5	R_3 =10

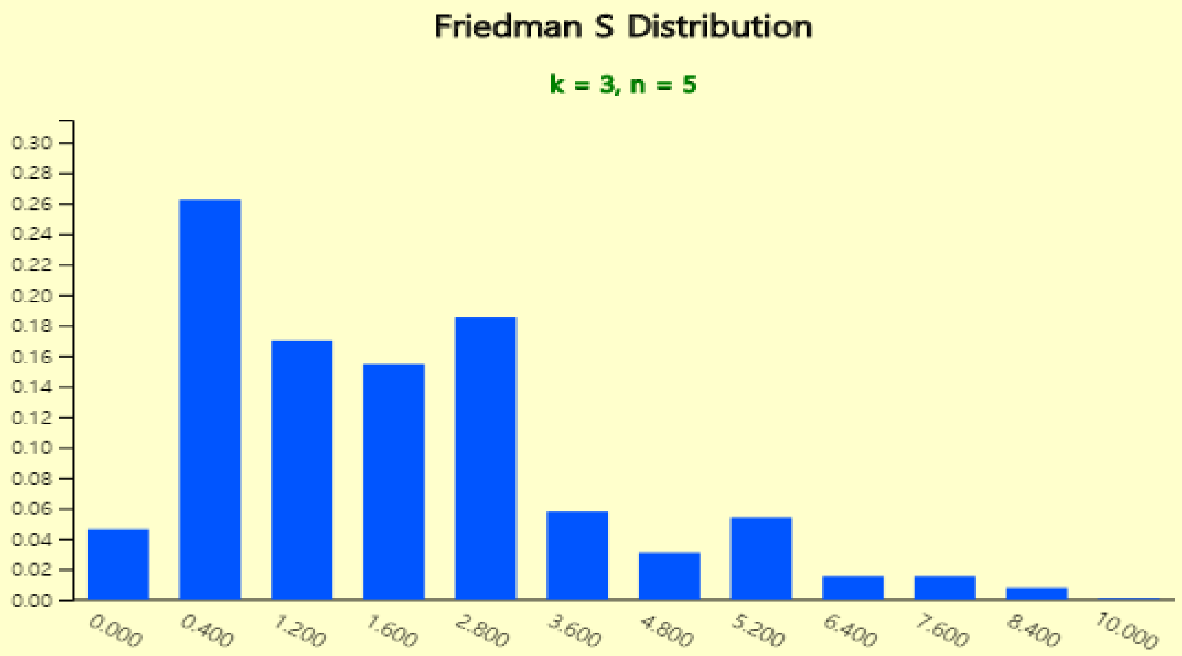
10.3 Nonparametric Test for Location of Several Populations

- Friedman Test Statistic

$$S = \frac{12}{nk(k+1)} \sum_{j=1}^{k} R_j^2 - 3n(k+1)$$

$$= \frac{12}{5\times3(3+1)} \left(15^2 + 5^2 + 10^2\right) - 3 \times 5(3+1) = 10$$

- If S > 6.4, then reject H_0 , hence reject H_0



Friedman S distribution	k = 3	n = 5	
x	P(X = x)	P(X≤x)	P(X≥x)
0.000	0.0463	0.0463	1.0000
0.400	0.2623	0.3086	0.9537
1.200	0.1698	0.4784	0.6914
1.600	0.1543	0.6327	0.5216
2.800	0.1852	0.8179	0.3673
3.600	0.0579	0.8758	0.1821
4.800	0.0309	0.9066	0.1242
5.200	0.0540	0.9606	0.0934
6.400	0.0154	0.9761	0.0394
7.600	0.0154	0.9915	0.0239
8.400	0.0077	0.9992	0.0085
10.000	0.0008	1.0000	0.0008

10.3 Nonparametric Test for Location of Several Populations

<Answer of 10.3.2>

Friedman Test Menu

[Hypothesis] $H_0: M_1 = M_2 = \dots = M_k$

H_1 : *At least one pair of locations is different*

[Test Type] Friedman Test

Significance Level α = ◉ 5% ○ 1%

[Sample Data] *(Treatment k = 3 or 4)*

Block 1 22.4 16.3 20.2

Block 2 16.1 12.6 15.2

Block 3 19.7 15.9 18.7

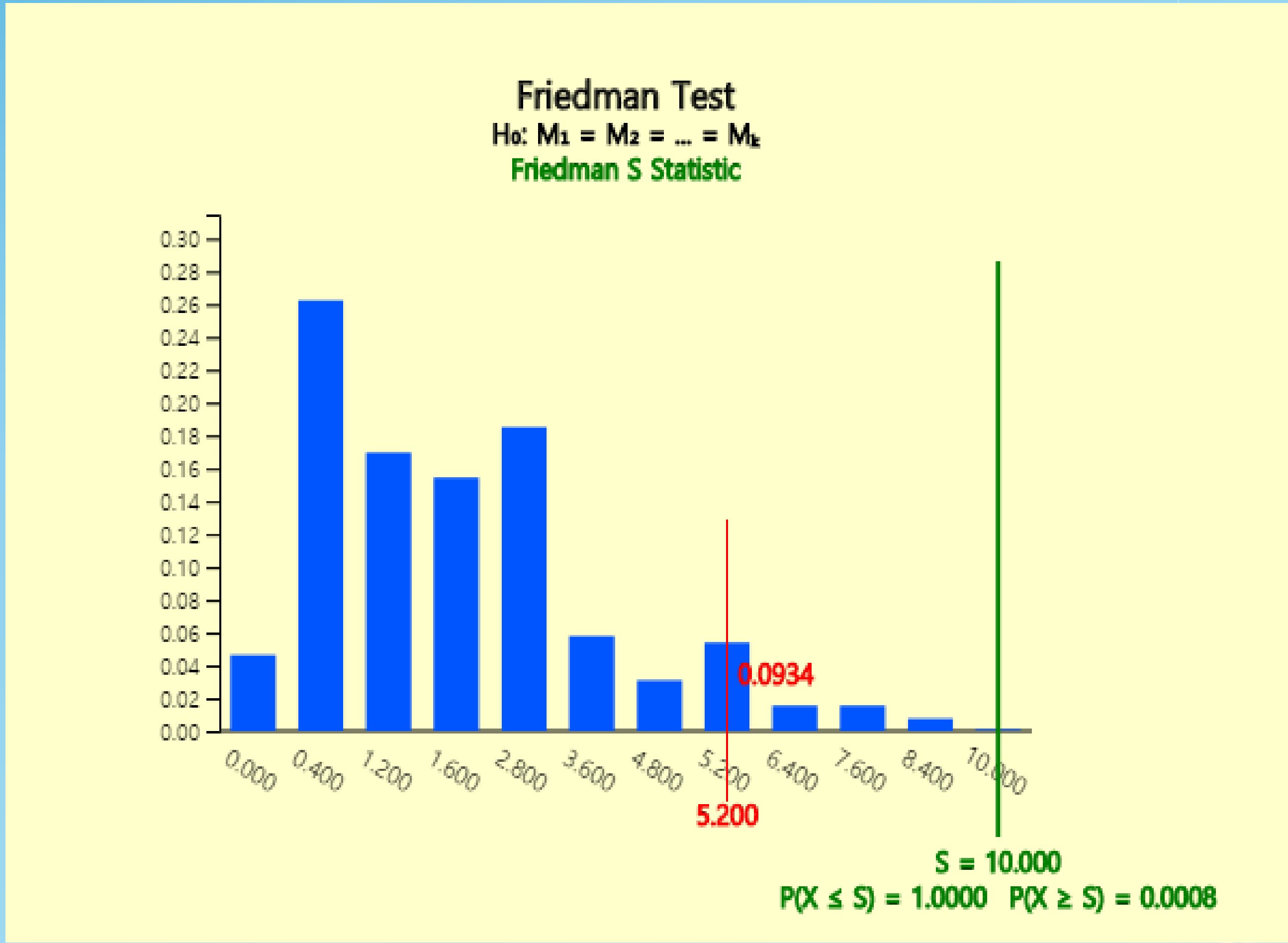
Block 4 21.1 17.8 18.9

Block 5 24.5 21.0 23.8

[Sample Statistics]

R_1 =	15	R_2 =	5	R_3 =	10	R_4 =	
k =	3	n =	5	S =	10.00		

Execute



10.3 Nonparametric Test for Location of Several Populations

10.3.2 Randomized Block Design : Friedman Test

Hypothesis	Decision Rule Test Statistic: S
$H_0 : \tau_1 = \tau_2 = \cdots = \tau_k$ H_1 : At least one pair of τ_j is different	If $S > s(k,n)_\alpha$, then reject H_0, else accept H_0

❖ $s(k,n)$: Friedman S distribution

❖ If there are tied values in each block, use the average of rank.

10.3 Nonparametric Test for Location of Several Populations

10.3.2 Randomized Block Design : Friedman Test

Table 10.3.13 Friedman Test – large sample case

Hypothesis	Decision Rule Test Statistic: S
$H_0 : \tau_1 = \tau_2 = \cdots = \tau_k$ H_1 : At least one pair of τ_j is different	If $S > \chi^2_{k-1;\alpha}$, then reject H_0, else accept H_0

❖ If there are tied values in each block, use the average of rank.

Thank you