Introduction to Statistics and Data Science using *eStat*

Chapter 11 Testing Hypothesis for Categorical Data

11.1.1 Goodness of Fit Test for Categorical Data

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11.1 Goodness of Fit Test

11.1.1 Goodness of Fit Test for Categorical Data
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11.1 Goodness of Fit Test

11.1.1 Goodness of Fit Test for Categorical Data

[Example 11.1.1] The result of a survey of 150 people before a local election to find out the approval ratings of three candidates is as follows.

- Looking at this frequency table alone, it seems that A candidate has a 40 percent approval rating, higher than the other candidates.
- Based on this sample survey, perform the goodness of fit test whether three candidates have the same approval rating or not. Use『eStatU』with the 5% significance level.

Candidate	Number of Supporters	Percent
A	60	40.0%
B	50	33.3%
C	40	25.7%
Total	150	100%

11.1 Goodness of Fit Test

<Answer of Example 11.1.1>

- **Hypothesis**

H_0 : Three candidates have the same approval rating. $(p_1 = p_2 = p_3 = \frac{1}{3})$

H_1: Three candidates have different approval rating.

- **Observed and Expected Frequency**

Candidate	Observed frequency (denoted as O_i)	Expected frequency (denoted as E_i)
A B C	O_1 = 60 O_2 = 50 O_3 = 40	E_1 = 50 E_2 = 50 E_3 = 50
Total	150	150

11.1 Goodness of Fit Test

<Answer of Example 11.1.1>

- **Test Statistic**

$$\chi^2_{obs} = \frac{(O_1 - E_1)^2}{E_1} + \frac{(O_2 - E_2)^2}{E_2} + \frac{(O_3 - E_3)^2}{E_3}$$
$$= \frac{(60 - 50)^2}{50} + \frac{(50 - 50)^2}{50} + \frac{(40 - 50)^2}{50} = 4$$

- **Decision Rule**

‘If $\chi^2_{obs} > \chi^2_{k-1;\,\alpha}$, reject H_0‘

Since $\chi^2_{3-1;\ 0.05} = 5.991$, H_0 cannot be rejected.

11.1 Goodness of Fit Test

<Answer of Example 11.1.1>

- **Confidence Interval**

A: $0.40 \pm 1.96\sqrt{\frac{0.40(1-0.40)}{150}} \Leftrightarrow (0.322, 0.478)$

B: $0.33 \pm 1.96\sqrt{\frac{0.33(1-0.33)}{150}} \Leftrightarrow (0.255, 0.405)$

C: $0.27 \pm 1.96\sqrt{\frac{0.27(1-0.27)}{150}} \Leftrightarrow (0.190, 0.330)$

11.1 Goodness of Fit Test

<Answer of Example 11.1.1>

Goodness of Fit Test

Menu

[Hypothesis] H_0 : *Observed & theoretical Distributions are the same*

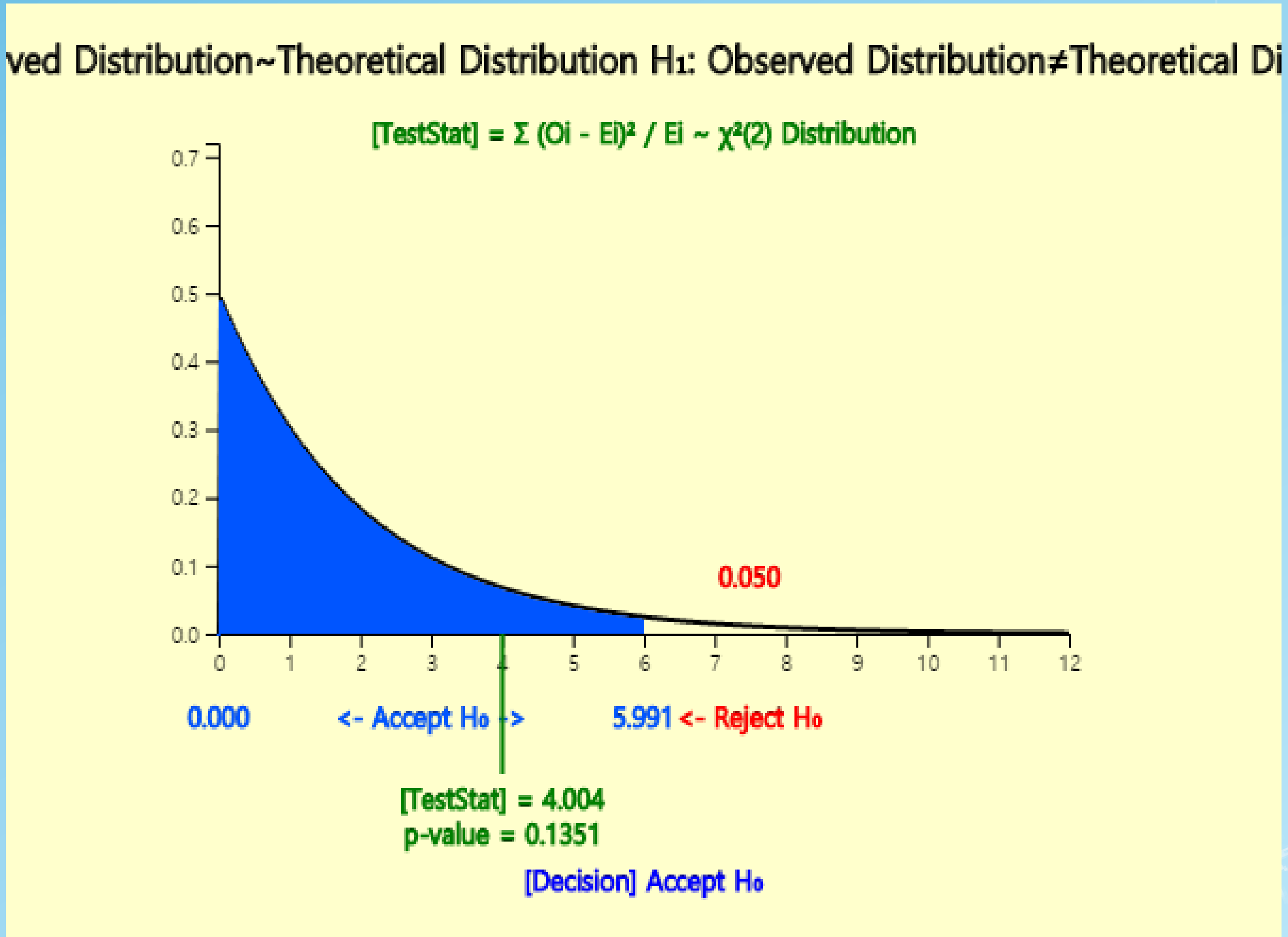
H_1 : *Observed & theoretical Distributions are different*

[Test Type] χ^2 *test* Significance Level α = ◉ 5% ○ 1%

[Sample Data] *Enter cell from upper left cell*

	Observed Frequency O	Expected Probability p	Expected Frequency E(>5)
Row 1	60	0.333	49.95
Row 2	50	0.333	49.95
Row 3	40	0.333	49.95
Row 4			
Row 5			
Row 6			
Row 7			
Row 8			
Row 9			
		합계	149.85

Execute



11.1 Goodness of Fit Test

[Goodness of Fit Test]

- A categorical variable X which has possible values $x_1, x_2, \ldots, x_k$ and their probabilities are $p_1, p_2, \ldots, p_k$ respectively.
- Observed frequencies from n samples are $(O_1, O_2, \ldots, O_k)$ and expected frequencies $(E_1, E_2, \ldots, E_k)$. The significance level is α.
- Hypothesis:

H_0: Distribution of $(O_1, O_2, \ldots, O_k)$ follows $(p_{10}, p_{20}, \ldots, p_{k0})$
H_1: Distribution of $(O_1, O_2, \ldots, O_k)$ do not follow $(p_{10}, p_{20}, \ldots, p_{k0})$

- Decision Rule:

'If $\chi^2_{obs} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} > \chi^2_{k-m-1;\,\alpha}$, reject H_0'

m is the number of population parameters estimated from the samples.
$*$ E_i should be greater than 5

Thank you