Introduction to Statistics and Data Science using *eStat*

Chapter 12 Correlation and Regression Analysis

12.1 Correlation Analysis

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12.1 Correlation Analysis

12.2 Simple Linear Regression Analysis

12.3 Multiple Linear Regression Analysis

12.1 Correlation Analysis

[Example 12.1.1] Based on the survey of advertising costs and sales for 10 companies that make the same product, we obtained the following data.

- Using 『eStat』, draw a scatter plot for this data and investigate the relation of the two variables.

Company	1	2	3	4	5	6	7	8	9	10
Advertise (X)	4	6	6	8	8	9	9	10	12	12
Sales (Y)	39	42	45	47	50	50	52	55	57	60

12.1 Correlation Analysis

<Answer of Example 12.1.1>

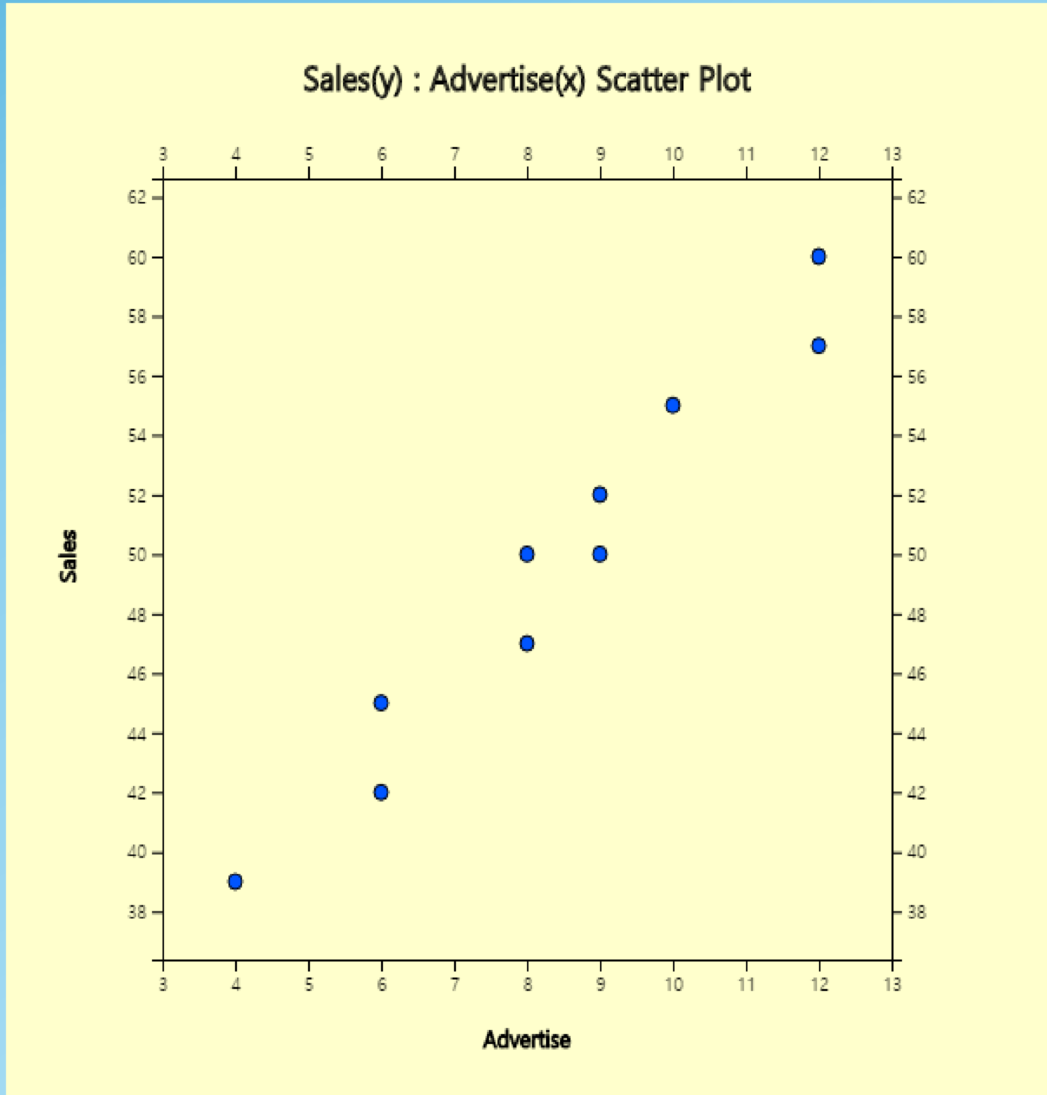
File: EX120101_SalesByAdvertise.csv

Y Var: 2: Sales by X Var: 1: Advertise

(Selected data: Raw Data) (Multiple Selection)

SelectedVar: V2 by V1,

	Advertise	Sales	V3	V4	V5
1	4	39			
2	6	42			
3	6	45			
4	8	47			
5	8	50			
6	9	50			
7	9	52			
8	10	55			
9	12	57			
10	12	60			
11					



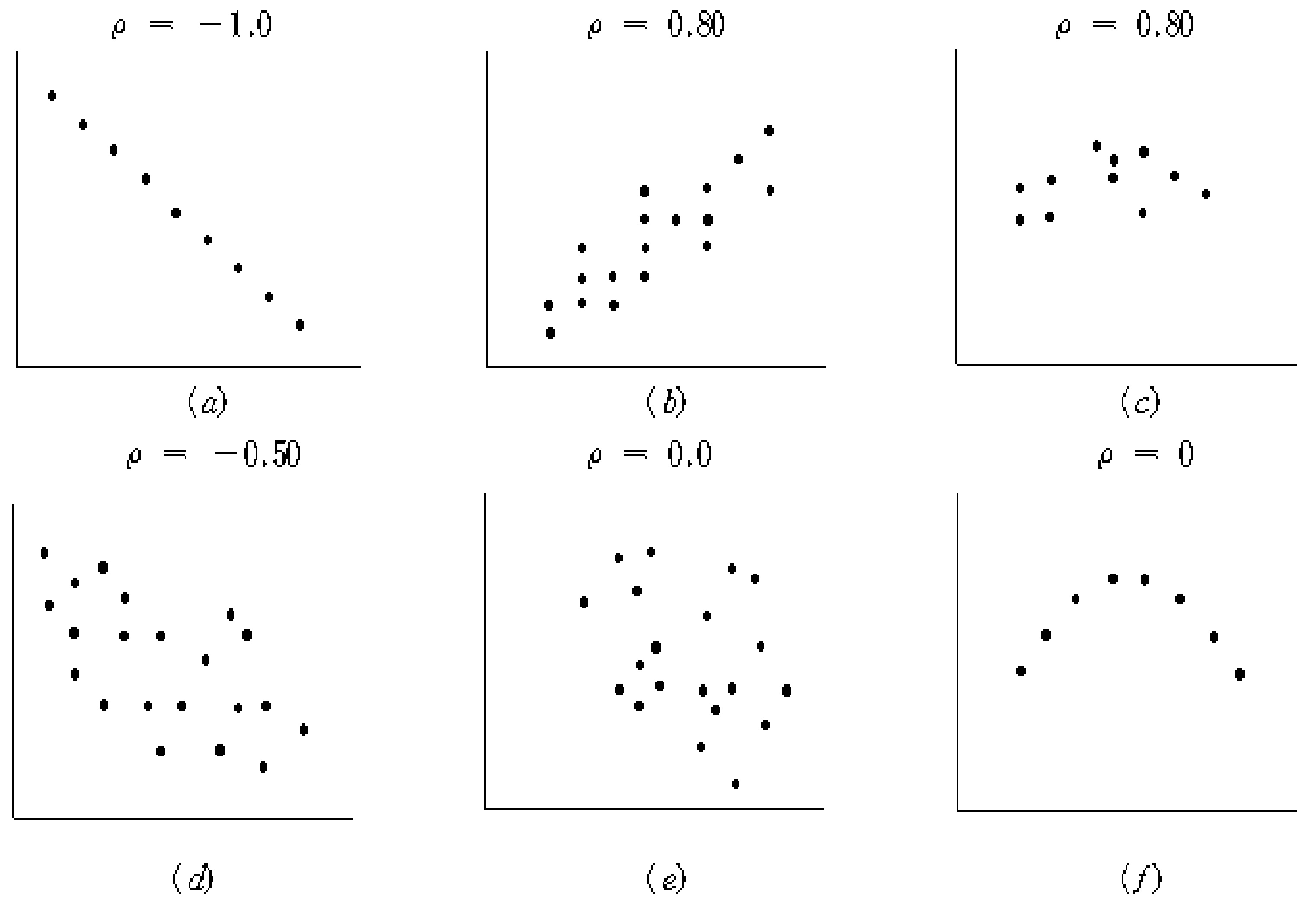
12.1 Correlation Analysis

- **Random Sample** $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$
  from a population with (μ_X, μ_Y) and (σ_X^2, σ_Y^2)

- **Population Covariance** $\sigma_{XY} = Cov(X, Y) = E(X_i - \mu_X)(Y_i - \mu_Y)$
- **Sample Covariance** $S_{XY} = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})$

- **Population Correlation** $\rho = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$
- **Sample Correlation** $r = \frac{S_{XY}}{S_X S_Y} = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2 \sum_{i=1}^{n}(Y_i - \bar{Y})^2}}$

12.1 Correlation Analysis



12.1 Correlation Analysis

- Characteristics of ρ

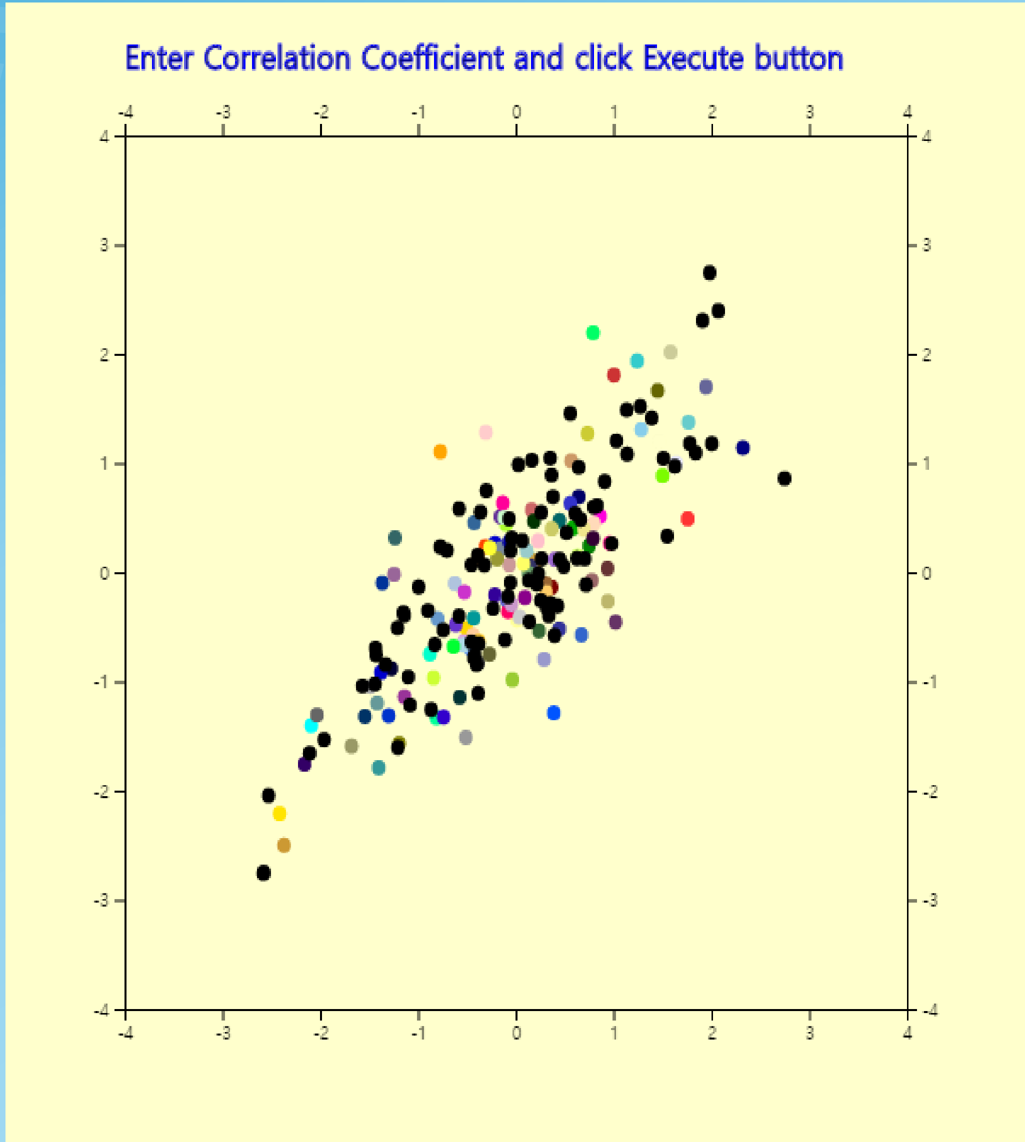
1) ρ has a value between -1 and +1.
   - closer to +1 ⇨ strong positive linear relation
   - closer to -1 ⇨ strong negative linear relation.
   - closer to 0 ⇨ weak linear relation

2) If all values of X and Y are located on a straight line, ρ is either +1 or -1.

3) ρ is only a measure of linear relationship between two variables.
   - if ρ =0, there is no linear relationship between the two variables, but there may be a different relationship

12.1 Correlation Analysis

- **Simulation of correlation coefficient**



12.1 Correlation Analysis

[Example 12.1.2] Find the sample covariance and correlation coefficient for the advertising costs and sales of [Example 12.1.1].

<Answer>

$$\mathrm{SXX} = \sum_{i=1}^{n}(X_i - \bar{X})^2 = \sum_{i=1}^{n} {X_i}^2 - n\,\bar{X}^2 = 766 - 10\times 8.4^2 = 60.4$$

$$\mathrm{SYY} = \sum_{i=1}^{n}(Y_i - \bar{Y})^2 = \sum_{i=1}^{n} {Y_i}^2 - n\,\bar{Y}^2 = 25097 - 10\times 49.7^2 = 396.1$$

$$\mathrm{SXY} = \sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y}) = \sum_{i=1}^{n} X_i Y_i - n\bar{X}\bar{Y} = 4326 - 10\times 8.4\times 49.7 = 151.2$$

id	X	Y	X^2	Y^2	XY
1	4	39	16	1521	156
2	6	42	36	1764	252
3	6	45	36	2025	270
4	8	47	64	2209	376
5	8	50	64	2500	400
6	9	50	81	2500	450
7	9	52	81	2704	468
8	10	55	100	3025	550
9	12	57	144	3249	684
10	12	60	144	3600	720
Sum	84	497	766	25097	4326
Mean	8.4	49.7			

12.1 Correlation Analysis

<Answer of Example 12.1.2>

$$s_{XY} = \frac{1}{n-1}\mathrm{SXY} = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y}) = \frac{151.2}{10-1}$$

$$r = \frac{s_{XY}}{s_X\, s_Y} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2 \sum_{i=1}^{n}(Y_i - \bar{Y})^2}} = \frac{SXY}{\sqrt{SXX\ \ SXY}} = \frac{151.2}{\sqrt{60.4\ \times 396.1}} = 0.978$$

12.1 Correlation Analysis

❑ Testing the population correlation coefficient ρ

Null Hypothesis: $H_0 : \rho = 0$

Test Statistic: $t_0 = \sqrt{n-2}\,\dfrac{r}{\sqrt{1-r^2}} \sim t_{n-2}$

Rejection Region of H_0 :

1) $H_1 : \rho < 0$ Reject H_0 if $t_0 < -t_{n-2;\ \alpha}$
2) $H_1 : \rho > 0$ Reject H_0 if $t_0 > t_{n-2;\ \alpha}$
3) $H_1 : \rho \neq 0$ Reject H_0 if $|\,t_0\,| > t_{n-2;\ \alpha/2}$

12.1 Correlation Analysis

[Example 12.1.3] In Example 12.1.2, test the hypothesis that the population correlation coefficient between advertising cost and the sales amount is zero at the significance level of 0.05.

<Answer>

$$t_0 = \sqrt{n-2}\ \frac{r}{\sqrt{1-r^2}} = \sqrt{10-2}\ \frac{0.978}{\sqrt{1-0.978^2}} = 13.26$$

$$t_{10-2;0.025} = 2.306$$

Hence $H_0 : \rho = 0$ is rejected

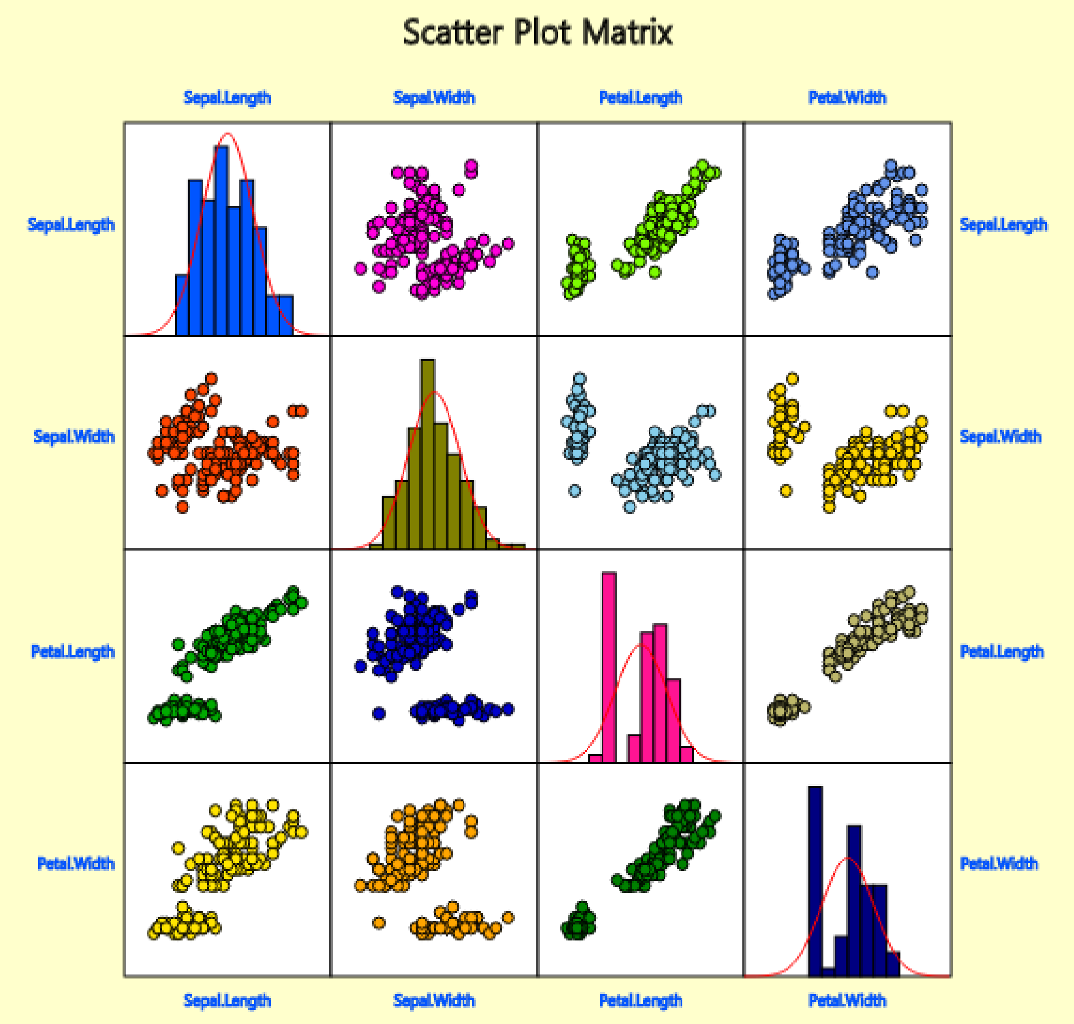
Regression Analysis				
Regression	y =	28.672 +	2.503 x	
Correlation Coefficient	r = 0.978	H_0: ρ = 0 H_1: ρ ≠ 0	t value = 13.117	p value < 0.0001
Coefficient of Determination	r^2 = 0.956			
Standard Error	s = 1.483			

12.1 Correlation Analysis

[Example 12.1.4] Draw a scatter plot matrix and correlation coefficient matrix using four variables of the iris data saved in the following location of『eStat』.

[Ex] ⇨ eBook ⇨ EX120104_Iris.csv

- **The variables are Sepal.Length, Sepal.Width, Petal.Length, and Petal.Width.**
- **Test the hypothesis whether the correlation coefficients are equal to zero.**



12.1 Correlation Analysis

<Answer of Example 12.1.4>

Descriptive Statistics

Variable	Variable Name	Observation	Mean	Std Dev	std err	95% Confidence Interval
Variable 1	Sepal.Length	150	5.843	0.828	0.068	(5.710, 5.977)
Variable 2	Sepal.Width	150	3.057	0.436	0.036	(2.987, 3.128)
Variable 3	Petal.Length	150	3.758	1.765	0.144	(3.473, 4.043)
Variable 4	Petal.Width	150	1.199	0.762	0.062	(1.076, 1.322)
Missing Observations	0					

Correlation Matrix

Correlation Analysis H_0: ρ=0 ρ≠0 t-value p-value	Variable Name	Variable 1	Variable 2	Variable 3	Variable 4
Variable 1	Sepal.Length	1	-0.118 t-value = -1.440 p-value 0.1519	0.872 t-value = 21.646 p-value < 0.0001	0.818 t-value = 17.296 p-value < 0.0001
Variable 2	Sepal.Width	-0.118 t-value = -1.440 p-value 0.1519	1	-0.428 t-value = -5.768 p-value < 0.0001	-0.366 t-value = -4.786 p-value < 0.0001
Variable 3	Petal.Length	0.872 t-value = 21.646 p-value < 0.0001	-0.428 t-value = -5.768 p-value < 0.0001	1	0.963 t-value = 43.387 p-value < 0.0001
Variable 4	Petal.Width	0.818 t-value = 17.296 p-value < 0.0001	-0.366 t-value = -4.786 p-value < 0.0001	0.963 t-value = 43.387 p-value < 0.0001	1

Thank you