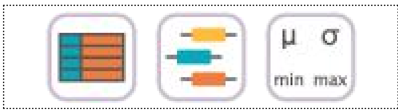
4

Data Summary Using Tables and Measures



SECTIONS

CHAPTER OBJECTIVES

Chapter 2 and 3 discussed how to visualize both the qualitative data and the quantitative data using graphs. Visualizing data using graphs makes easy and fast to see any information that is nested in data. However, if you want more detailed information, it is better to summarize data by using tables or measures.

In section 4.1, we introduce a frequency table as a summary of single variable.

In section 4.2, we introduce a contingency table as a summary of two variables.

In section 4.3 we "introduce measures to summarize the quantitative data and a box plot.

4.1 Frequency Table for Single Variable

- A **frequency table** of qualitative data summarizes frequencies of each possible value of a categorical variable. A frequency table is the most commonly used tool to summarize qualitative data. The frequency table also shows relative frequencies (percents) which are calculated by dividing the frequency of each category with the number of observations belong to the category, and cumulative relative frequencies accumulated in the order of the categories. The bar graph, the pie chart and the band graph in Chapter 2 are drawn by using this frequency table of qualitative data.
- The frequency table is usually used to summarize qualitative data, but it can also be used to summarize quantitative data by transforming it to qualitative data. All possible values of the quantitative data are divided into several intervals which are not overlapped with each other and the number of observations belong to each interval is counted to make a frequency table.

Definition

Frequency Table

A **frequency table** of qualitative data summarizes frequencies of each possible value of a categorical variable.

The frequency table can also be used to summarize quantitative data by transforming it to qualitative data. All possible values of the quantitative data are divided into several intervals which are not overlapped with each other and the number of observations belong to each interval is counted to make a frequency table.

- A frequency table of sample data can be used to test the goodness of fit of data whether data follow a particular distribution as described in Chapter 11.

4.1.1 Frequency Table for Categorical Variable

Example 4.1.1

(Gender Raw Data)

In Example 2.3.1, a bar graph of the gender variable in a class was drawn by using the raw data shown in Table 4.1.1. The bar graph was able to be drawn by using the frequencies of male and female students. Use 『eStat』 to create a frequency table for this raw data of the gender variable.

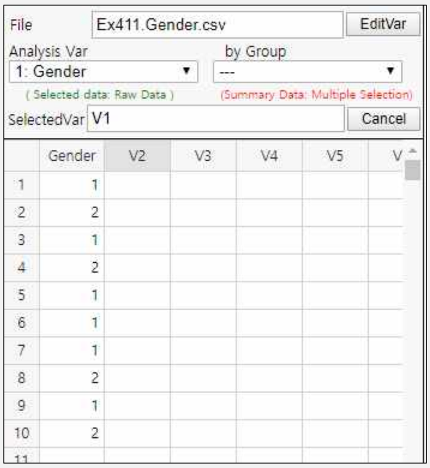
Table 4.1.1 Raw data of the gender (1:male, 2: female)

Gender
1
2
1
2
1
1
1
2
1
2

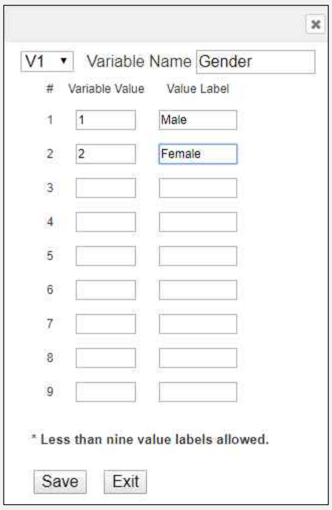
Ex ⇨ eBook ⇨ EX040101_Categorical_Gender.csv.

Example 4.1.1 Answer

- Enter the gender data of Table 4.1.1 to 『eStat』 as in <Figure 4.1.1>. Use [Edit Var] button to enter the variable name ‘Gender’ and its value labels as 1 for ‘Male’ and 2 for ‘Female’ as in <Figure 4.1.2>. The data that were edited for their value labels must be saved in JSON format (click on the icon) to ensure that the entered information is not lost. When you load a file in JSON format, you must also use the JSON Open icon which is for opening a file in JSON format.

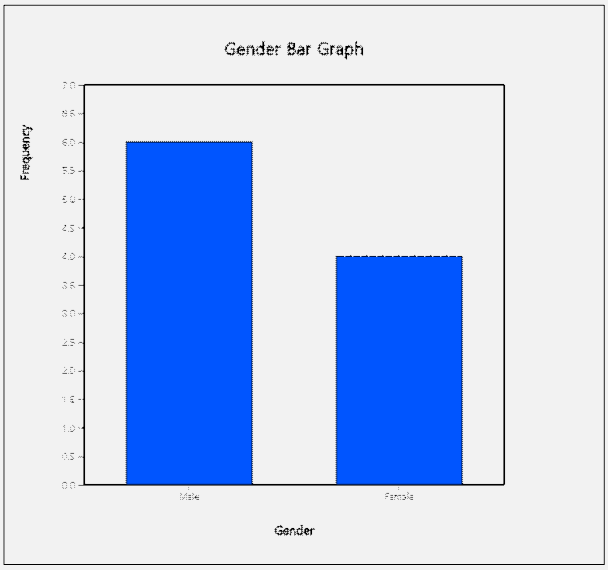


<Figure 4.1.1> Input gender data of a class



<Figure 4.1.2> Input variable name and value label

- If you select the gender variable as the 'Analysis Var' in the variable selection box as shown in <Figure 4.1.1>, a bar graph of the gender is drawn as in <Figure 4.1.3>. Then, if you click the Frequency Table icon , the frequency table of the gender variable will appear in the Log Area, as in <Figure 4.1.4>. This frequency table is used to draw the bar graph or the pie chart.



<Figure 4.1.3> Bar graph of the gender

Example 4.1.1 Answer (continued)

Frequency Table	Analysis Var	(Gender)		
Var Value	Value Label	Frequency	Relative Frequency (%)	Cumulated Relative Frequency (%)
1	Male	6	60.0	60.0
2	Female	4	40.0	100.0
Total		10	100.0	
	Missing Observations	0		

<Figure 4.1.4> Frequency table of the gender

[Practice 4.1.1]

(Vegetable Preference)

Data that examined gender (1: male, 2: female) and vegetable preference(1: lettuce, 2: spinach, 3: pumpkin, 4: eggplant) of an elementary school class can be found at the following location of『eStat』.

⇨ eBook ⇨ PR040101_Categorical_VegetablePrefByGender.csv.

By using『eStat』, find a frequency table of the vegetable preference.

4.1.2 Frequency Table for Quantitative Variable

- The quantitative data can have too many possible values and a frequency table of the quantitative data may not be easy to analyze. In order to make a frequency table for quantitative data which can be analyzed easily, possible values of the data are divided into several intervals and frequencies of each interval are investigated. Generally, the intervals are not overlapped with each other and the number of data in each interval is counted. For this purpose, the maximum and the minimum of data are first investigated to calculate the range of the data and then determine the number of intervals. The number of intervals is typically between 5 and 10, but it may depend on a researcher’s choice. Some researchers prefer to use the square root of the number of observations. If the number of intervals is determined, the range of data (maximum - maximum) is divided by the number of intervals to calculate the width of the interval. Starting and ending points of each interval are usually described as ‘from greater than or equal ($\geq$) a 'to less than (<) b’ which means a one-sided close interval $[a, b)$.

Example 4.1.2

(Otter length)

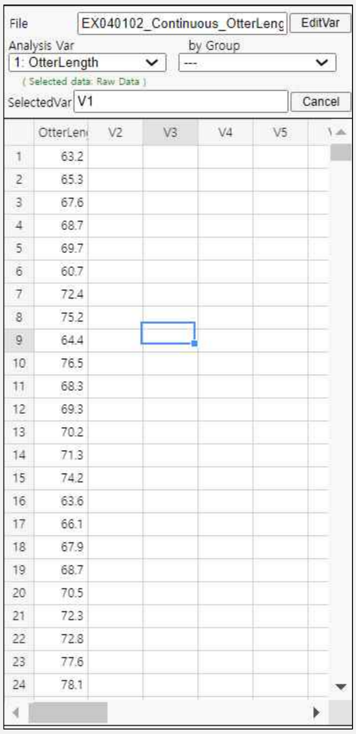
Data of 30 otter lengths can be found at the following location of『eStat』.

⇨ eBook ⇨ EX040120_Continuous_OtterLength.csv.

Draw a histogram and frequency table of the otter lengths by using『eStat』.

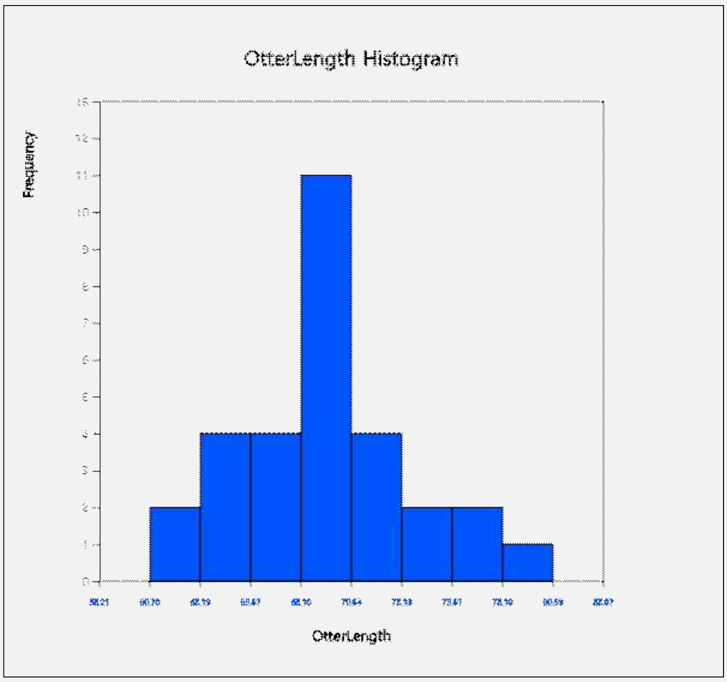
Example 4.1.2 Answer

- Retrieve the data from『eStat』as in <Figure 4.1.5>.



<Figure 4.1.5> Data of Otter Length

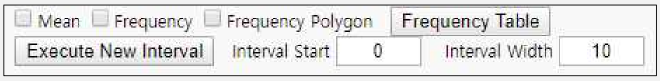
- Click the Histogram Icon and then select the variable name 'OtterLength' to draw a histogram as shown in <Figure 4.1.6>.



<Figure 4.1.6> Histogram of the otter length

Example 4.1.2 Answer (continued)

- Click on the [Frequency Table] button in the options window below the histogram (<Figure 4.1.7>). Then a frequency table of the histogram intervals is shown as in <Figure 4.1.8> in the Log Area.

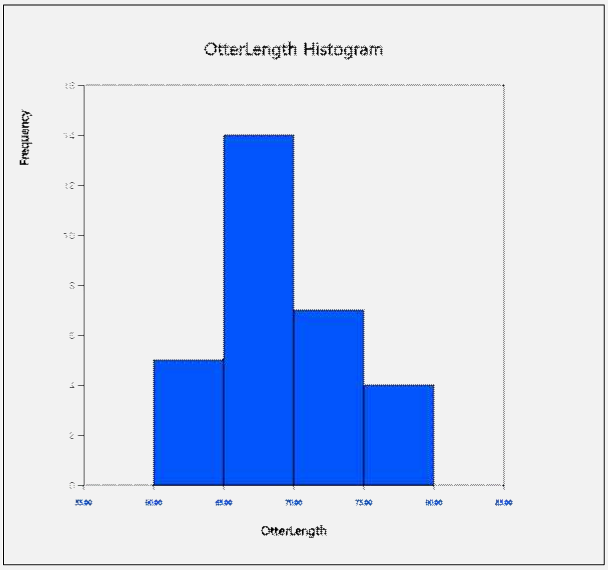


<Figure 4.1.7> Options of the histogram

Histogram Frequency Table	Group Name	0
Interval (OtterLength)	Group 1 (null)	Total
1 [60.70, 63.19)	2 (6.7%)	2 (6.7%)
2 [63.19, 65.67)	4 (13.3%)	4 (13.3%)
3 [65.67, 68.16)	4 (13.3%)	4 (13.3%)
4 [68.16, 70.64)	11 (36.7%)	11 (36.7%)
5 [70.64, 73.13)	4 (13.3%)	4 (13.3%)
6 [73.13, 75.61)	2 (6.7%)	2 (6.7%)
7 [75.61, 78.10)	2 (6.7%)	2 (6.7%)
8 [78.10, 80.59]	1 (3.3%)	1 (3.3%)
Total	30 (100%)	30 (100%)

<Figure 4.1.8> Frequency table of histogram for otter length

- If you want to adjust the histogram intervals from 60kg with an interval length of 5kg, set ‘Interval Start’ to 60 and ‘Interval Width’ to 5 in the graph options. Press [Execute New Interval] button to display the adjusted histogram as shown in <Figure 4.1.9>. Click on [Frequency Table] button to reveal a new frequency table as in <Figure 4.1.10>.



<Figure 4.1.9> Adjusted histogram of otter length

Example 4.1.2 Answer (continued)

Histogram Frequency Table	Group Name	()
Interval (OtterLength)	Group 1 (null)	Total
1 [60.00, 65.00)	5 (16.7%)	5 (16.7%)
2 [65.00, 70.00)	14 (46.7%)	14 (46.7%)
3 [70.00, 75.00)	7 (23.3%)	7 (23.3%)
4 [75.00, 80.00)	4 (13.3%)	4 (13.3%)
Total	30 (100%)	30 (100%)

<Figure 4.1.10> Adjusted frequency table of the otter length

[Practice 4.1.2]

(Age of Library Visitors)

The following data is a survey on the age of 30 people who visited a library in the morning. Draw an appropriate histogram and its frequency table using『eStat』.

28 55 26 35 43 47 47 17 35 36 48 47 34 28 43
20 30 53 27 32 34 43 18 38 29 44 67 48 45 43

⇨ eBook ⇨ PR040102_Continuous_LibraryVisitorAge.csv.

4.2 Contingency Table for Two Variables

- A **contingency table** or **cross table** is used to summarize two categorical variables and is also used to study an association of two variables. A cross table divides a table into rows and columns to create cells by using possible values of two categorical variables, and then counts the number of observations (frequency) belonging to the corresponding cell. Percentage of each cell for the sum of rows, or percentage of each cell for the sum of columns can be shown in a contingency table for further analysis. Percentage of each cell for the total number of data can also be shown in a cross table.
- A contingency table is usually made for two qualitative data. In case of two quantitative data, the quantitative data can be transformed into qualitative data by using intervals, and then a contingency table for these qualitative data can be created.

Definition

Contingency Table

A **contingency table or cross table** divides a table into rows and columns to create cells by using possible values of two categorical variables, and then counts the number of observations (frequency) belonging to the corresponding cells.

In case of two quantitative data, the data can be transformed into qualitative data by using intervals, and then a contingency table for these qualitative data can be created.

- If we examine frequencies of a contingency table, it is possible to check an association between two variables. We will discuss in detail about statistical analysis of a cross table such as independence test or homogeneity test in Chapter 11.

4.2.1 Contingency Table for Two Categorical Variables

- Let us discuss how to create a contingency table from the raw data of two categorical variables using the following example.

Example 4.2.1

(Survey on Gender and Marital Status)

Table 4.2.1 shows survey data on gender (1: Male, 2: Female) and marital status (1: Single, 2: Married, 3: Other) which are used in Example 2.2.3. Create a contingency table of the marital status by gender using 『eStat』.

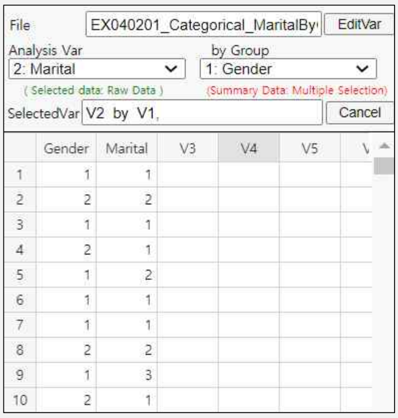
Table 4.2.1 Survey data on gender and marital status

Gender	Marital Status
1	1
2	2
1	1
2	1
1	2
1	1
1	1
2	2
1	3
2	1

Ex ⇨ eBook ⇨ EX040201_Categorical_MaritalByGender.csv.

Example 4.2.1 Answer

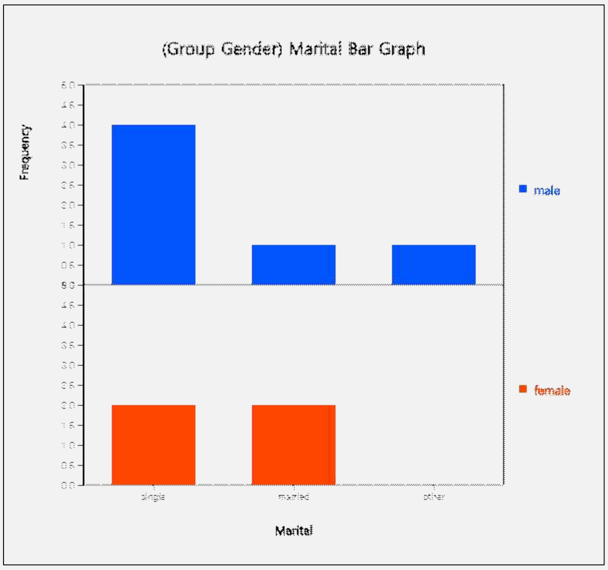
- Enter the data of the gender and the marital status in Table 4.2.1 to the sheet of 『eStat』 as in <Figure 4.2.1>. Use [Edit Var] button to enter a variable name 'Gender' and value labels 'Male' for 1 and 'Female' for 2. In the same way, enter a variable name 'Marital' and value labels 'Single' for 1, 'Married' for 2 and 'Other' for 3. The data that were edited for their value labels should be saved in JSON format file by clicking on the icon. If you want to load this file in JSON format, you must also click on the icon which is for loading a file in JSON format.



<Figure 4.2.1> Data input on gender and marital status

Example 4.2.1 Answer (continued)

- Click on the variable name 'Marital' ('Analysis Var'), and then the variable name 'Gender' ('by Group'). Then you will see a bar graph of the marital status by gender as in <Figure 4.2.2> which is a default graph. Click the Frequency Table icon to display a contingency table of the marital status by gender in the Log Area as in <Figure 4.2.3>. In this contingency table, the 'by Group' variable becomes the row variable and the 'Analysis Var' becomes the column variable. This contingency table was used to draw the bar graph of the marital status by gender as in <Figure 4.2.2>.



<Figure 4.2.2> Bar graph on marital status by gender

Cross Table	Col Variable	(Marital)			
Row Variable (Gender)	single	married	other	Total	
male	4 66.7%	1 16.7%	1 16.7%	6 100%	
female	2 50.0%	2 50.0%	0 0.0%	4 100%	
Total	6 60.0%	3 30.0%	1 10.0%	10 100%	
	Missing Observations	0			
Independence Test					
Sum of χ^2 value	1.667	deg of freedom	2	p-value	0.4346

<Figure 4.2.3> Contingency table on marital status and gender

[Practice 4.2.1]

(Survey on Gender and Vegetable Preference)

In a class of an elementary school, a survey on gender (1: male, 2: female) and favorite vegetable (1: lettuce, 2: spinach, 3: pumpkin, 4: eggplant) was conducted. The survey data can be found at the following location of『eStat』.

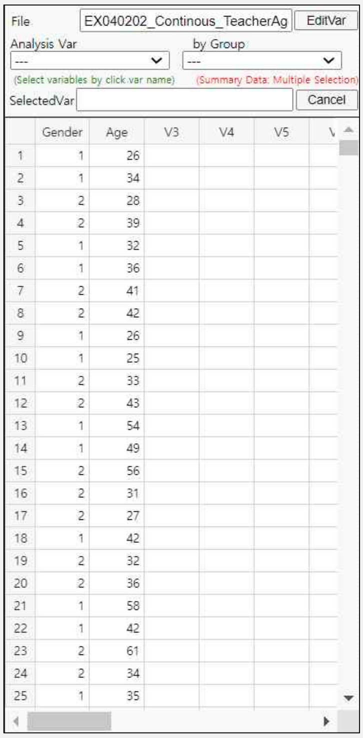
Ex ⇨ eBook ⇨ PR040201_Categorical_VegetablePrefByGender.csv.

Create a contingency table of the favorite vegetable by gender.

4.2.2 Contingency Table for Two Quantitative Variables

- In order to create a contingency table for two quantitative variables, we need to divide all possible values of each quantitative variable into some number of intervals as we did when creating a frequency table of single quantitative variable.
- If both variables are quantitative, it is advisable to use a statistical software such as R, SPSS, and SAS etc. If one variable is categorical and the other one is quantitative, then a contingency table can be made by using 『eStat』. Let's take a look at the following example.

Example 4.2.2	**(Survery on Teacher's Age and Gender)** In a middle school, a survey on gender and age of all teachers was conducted. The suvey data can be found at the following location of 『eStat』. Ex ⇨ eBook ⇨ EX040202_Continuous_TeacherAgeByGender.csv. By using the histogram module of 『eStat』, create a contingency table of the age by gender.
Example 4.2.2 Answer	◆ Retrieve the data from 『eStat』 as in <Figure 4.2.4> and enter value labels of 'Gender' as 'Male' for 1 and 'Female' for 2.

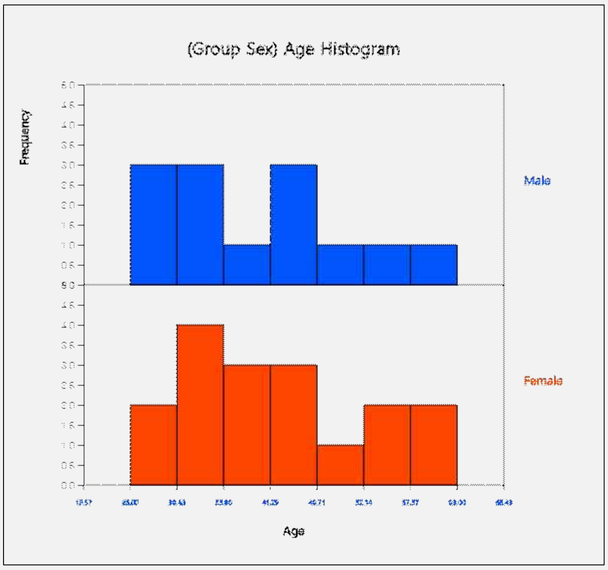


	Gender	Age	V3	V4	V5
1	1	26			
2	1	34			
3	2	28			
4	2	39			
5	1	32			
6	1	36			
7	2	41			
8	2	42			
9	1	26			
10	1	25			
11	2	33			
12	2	43			
13	1	54			
14	1	49			
15	2	56			
16	2	31			
17	2	27			
18	1	42			
19	2	32			
20	2	36			
21	1	58			
22	1	42			
23	2	61			
24	2	34			
25	1	35			

<Figure 4.2.4> Data input on gender and age

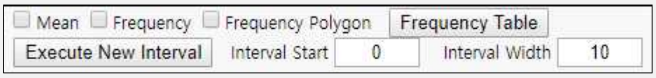
◆ After clicking the histogram icon, select the 'Age' variable as 'Analysis Var', and then the 'Gender' variable as 'by Group'. A histogram will appear as shown in <Figure 4.2.5>.

Example 4.2.2 Answer (continued)



<Figure 4.2.5> Histogram on age by gender

- If you click the button of 'Frequency Table' in the options window below the graph (<Figure 4.2.6>), a contingency table will appear in the Log Area as shown in <Figure 4.2.7>.



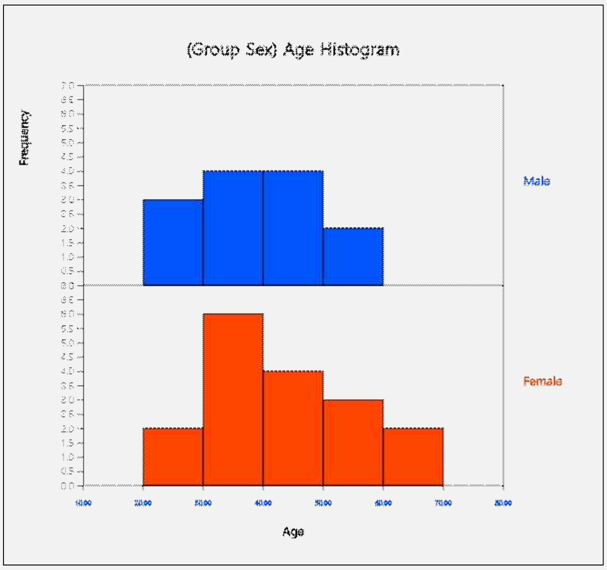
<Figure 4.2.6> Options of the histogram

Histogram Frequency Table	Group Name	(Gender)	
Interval (Age)	Group 1 (Male)	Group 2 (Female)	Total
1 [25.00, 30.43)	3 (23.1%)	2 (11.8%)	5 (16.7%)
2 [30.43, 35.86)	3 (23.1%)	4 (23.5%)	7 (23.3%)
3 [35.86, 41.29)	1 (7.7%)	3 (17.6%)	4 (13.3%)
4 [41.29, 46.71)	3 (23.1%)	3 (17.6%)	6 (20.0%)
5 [46.71, 52.14)	1 (7.7%)	1 (5.9%)	2 (6.7%)
6 [52.14, 57.57)	1 (7.7%)	2 (11.8%)	3 (10.0%)
7 [57.57, 63.00)	1 (7.7%)	2 (11.8%)	3 (10.0%)
Total	13 (100%)	17 (100%)	30 (100%)

<Figure 4.2.7> Contingency table of age by gender

Example 4.2.2 Answer (continued)

- If the intervals of the histogram in <Figure 4.2.5> are to be readjusted, for example, from 20 to 10 years apart, set 'Interval Start' to 20 and 'Interval Width' to 10 in the graph options and press [Execute New Interval] button. Then a histogram with the adjusted intervals is appeared as in <Figure 4.2.8>, and a contingency table with the adjusted intervals can be obtained by clicking on [Frequency Table] button as shown in <Figure 4.2.9>.



<Figure 4.2.8> Histogram with adjusted intervals

Histogram Frequency Table	Group Name	(Sex)	
Interval (Age)	Group 1 (Male)	Group 2 (Female)	Total
1 [20.00, 30.00)	3 (23.1%)	2 (11.8%)	5 (16.7%)
2 [30.00, 40.00)	4 (30.8%)	6 (35.3%)	10 (33.3%)
3 [40.00, 50.00)	4 (30.8%)	4 (23.5%)	8 (26.7%)
4 [50.00, 60.00)	2 (15.4%)	3 (17.6%)	5 (16.7%)
5 [60.00, 70.00)	0 (0.0%)	2 (11.8%)	2 (6.7%)
Total	13 (100%)	17 (100%)	30 (100%)

<Figure 4.2.9> Contingency table with adjusted intervals

[Practice 4.2.2]

(Oral Cleanliness by Brushing Method)

Data of oral cleanliness score according to brushing methods (1: basic method, 2: rotation method) can be found at the following location of 『eStat』.

Ex ⇨ eBook ⇨ PR040202_Continuous_ToothCleanByBrushMethod.csv.

Create a contingency table of oral cleanliness by brushing method.

4.3 Summary Measures for Quantitative Variable

- The quantitative data can be summarized by using measures of central tendancy in section 4.3.1 and measures of dispersion in 4.3.2.

4.3.1 Measures of Central Tendency

- Average, median and mode are the most frequently used measures of central tendency to summarize the quantitative data.
- A **mean or average** is the sum of all data values divided by the number of data. If n is the number of data and the data values are denoted as $x_1, x_2, ..., x_n$, the mean is defined as follows:

$$Mean \;=\; \frac{x_1 + x_2 + ... + x_n}{n} \;=\; \frac{1}{n}\sum_{i=1}^{n} x_i$$

- The mean can be understood as the center of gravity representing data. Therefore, the sum of deviations which subtracts mean from each data is zero as follows:

$$\sum_{i=1}^{n}(x_i - Mean) \;=\; 0$$

- If data are from a population, the mean of this data is referred to as a **population mean** and is usually denoted as μ in Greek letter. If data are sampled from a population, the mean of this data is referred as a **sample mean** and denoted as $\overline{x}$ (read as 'x bar'). The sample mean has many good characteristics (Chapter 6) and is frequently used to estimate the population mean. Note that the mean is heavily influenced by an extreme point where one data value is far from the data cluster.
- A **median** is the value placed in the middle when data are listed in ascending order of their values and is denoted as m if data are sampled from a population or M if data are from a population. If the number of data, n, is an odd number, the median is the data value located at the $((n+1)/2)^{th}$ when data are arranged in ascending order. If n is an even number, then the median is the average of the data values located at the $(n/2)^{th}$ and $((n/2)+1)^{th}$.

$$Median \;=\; \begin{cases} (\frac{n+1}{2})^{th}\ data & \text{if } n\ is\ odd \\ Average\ of\ (\frac{n}{2})^{th,}\ (\frac{n}{2}+1)^{th} & \text{if } n\ is\ even \end{cases}$$

- The median is not sensitive even if there is an extreme point in data, so it is often used as a measure of the central tendency when there is an extreme point.
- A **mode** is the most frequently occurred value among data values.

$$Mode \;=\; the\ most\ frequently\ occurred\ value\ among\ data\ values$$

- In case of the quantitative data, since there might be so many possible values, it is not reasonable to set a mode value as the most frequently occurred data value. In this case, we usually transform the quantitative data into the qualitative data by dividing the data values into several not-overlapped intervals and count frequencies of each interval. The middle value of an interval which has the highest frequency is set to the mode.

Definition	**Mean, Median and Mode**
	A **mean or average** is the sum of all observed data divided by the number of data. The mean can be understood as the center of gravity representing data. The population mean is denoted as μ and the sample mean is denoted as $\overline{x}$. A **median** is the value placed in the middle when data are listed in ascending order of their values. The population median is usually denoted as M and the sample median is denoted as m. A **mode** is the most frequently occurred value among data values.

Example 4.3.1

(Quiz scores)

Quiz scores of seven students in a class of Statistics are sampled randomly as follows:

5, 6, 3, 7, 9, 4, 8

Ex ⇨ eBook ⇨ EX040301_Continuous_QuizScore.csv.

Calculate the mean and median of this data and compare the result with 『eStat』 output.

Example 4.3.1 Answer

- The sample mean is calculated as follows:

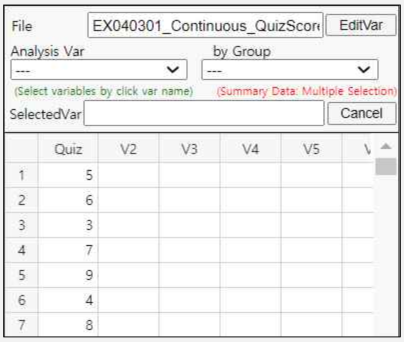
$\overline{x}$ = (5 + 6 + 3 + 7 + 9 + 4 + 8) / 7 = 6

- In order to find the sample median, first arrange the data in ascending order of data values as follows:

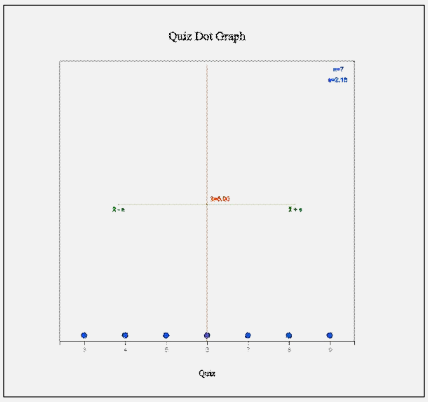
3, 4, 5, 6, 7, 8, 9.

Since the sample size, 7, is an odd number, median is $(\frac{n+1}{2})^{th} = (\frac{7+1}{2})^{th} = 4^{th}$ data in the arranged data as above which is 6.

- In order to use 『eStat』 , enter the data in column V1 of the sheet as in <Figure 4.3.1>. Click the Dot Graph icon and click the variable name 'Quiz' to see the dot graph of data as in <Figure 4.3.2>. If you check the option 'Mean/StdDev', you can see the location of mean and the length of standard deviation.



<Figure 4.3.1> Data input



<Figure 4.3.2> Dot graph with mean and standard deviation.

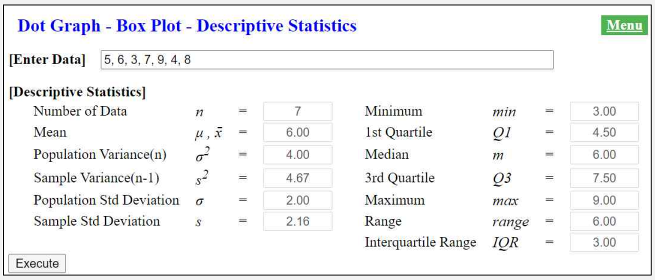
- If you click the Descriptive Statistics icon, then a table of all descriptive statistics will result in the Log Area as shown in <Figure 4.3.3>. It shows not only mean and median, but also other statistics such as the standard deviation, minimum, and maximum etc.

Example 4.3.1 Answer (continued)

Descriptive Statistics	Analysis Var (Quiz)
Observation	7
Missing Observations	0
Mean	6.000
Variance (n)	4.000
Variance (n-1)	4.667
Std Dev (n)	2.000
Std Dev (n-1)	2.160
Minimum	3.000
1st Quartile	4.500
Median	6.000
3rd Quartile	7.500
Maximum	9.000
Range	6.000
Interquartile Range	3.000
Coefficient of Variation (n)	33.33 %
Coefficient of Variation (n-1)	36.00 %

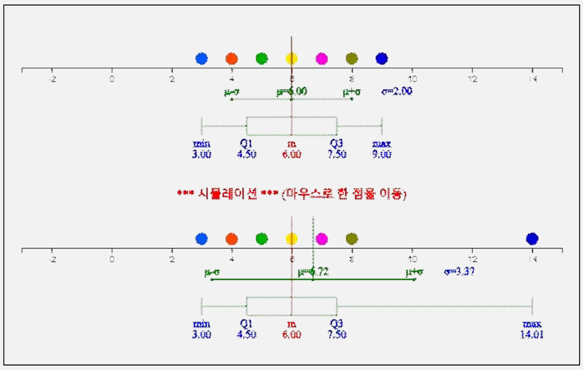
<Figure 4.3.3> Basic statistics of data

- You can also use 『eStatU』 to calculate the descriptive statistics and simulate an influence of extreme point. Select ‘Dot Graph – Box Plot – Descriptive Statistics’ from the menu of 『eStatU』 and enter data as in <Figure 4.3.4>. 『eStatU』 calculates all statistics while you are entering data.



<Figure 4.3.4> 『eStatU』 basic statistics of data

- If you click the [Execute] button, two sets of dot graph and box plot appear as in <Figure 4.3.5>. The first graph is for the data you entered and the second one is for simulation. On the second bar graph of <Figure 4.3.5>. you can click a dot using your mouse and move to other location of axis (make an extreme point) to check its influence on mean and median. You can see that the mean is changed a lot by the extreme point, but the median is not changed by the extreme point.



<Figure 4.3.5> 『eStatU』 simulation of an extreme point

[Practice 4.3.1]

(Otter Length)

The lengths of 30 otters are measured (in cm) and the data are saved at the following location of 『eStat』.

⇨ eBook ⇨ PR040301_Continuous_OtterLength.csv.

1) Use 『eStat』 to obtain the mean, median, minimum and maximum of this data.
2) Copy this data to 『eStatU』 and draw a dot graph and a box plot. Simulate the influence of an outlier.

Example 4.3.2

(Library Visitor)

If a frequency table of visitors' age in a library is as shown in Table 4.3.1, find the mode of the age based on this frequency table.

Table 4.3.1 Frequency table of visitor's age in a libray

Age Interval	Frequency (%)
[20.00, 30.00)	2 (6.7%)
[30.00, 40.00)	7 (23.3%)
[40.00, 50.00)	7 (23.3%)
[50.00, 60.00)	9 (30.0%)
[60.00, 70.00)	3 (10.0%)
[70.00, 80.00)	2 (6.7%)
Total	30 (100%)

Answer

- The interval [50.00, 60.00) has the highest frequency which is 9 and median is the mid value of the interval [50.00, 60.00) is 55.

- There are several variants to compensate the disadvantage of the mean, one of which is a **trimmed mean**. This is to list the data in order and then average the data except for a constant number of large and small values respectively in order to eliminate the extremes. The trimmed mean is often used to prevent biased judging by referees in sports such as gymnastics and figure skating at the Olympics. You may remove the top few percent data instead of the maximum and the bottom few percent data instead of the minimum.
- Another variant is a **weighted mean** in which each measurement is multiplied by a constant weight to obtain the mean. The grade point average for college students which uses the weights of credit hours is an example of the weighted mean. The price index which uses the weights of the total amount of sales of the goods is another example of the weighted mean. If $x_1, x_2, \cdots, x_n$ are the data values and their weights are $w_1, w_2, \cdots, w_n$, then the weighted mean is defined as the following.

$$Weighted\ Mean = \frac{w_1 x_1 + w_2 x_2 + \cdots + w_n x_n}{w_1 + w_2 + \cdots + w_n} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

Definition	**Trimmed Mean and Weighted Mean** A **trimmed mean** is the average of data except for a constant number of large and small values respectively in order to eliminate extremes. A **weighted mean** is the average of weighted sum in which each measurement is multiplied by some weight and divided by the sum of all weights.

Example 4.3.3	**(Olympic Gymnastics Game)** An Olympic Gymnastics Game was judged by eight referees and their result are as follows: 9.0 9.5 9.3 7.2 10.0 9.1 9.4 9.0 Find the mean and median of this data. Also, find the trimmed mean which excludes the minimum and the maximum. Compare both results.
Answer	◆ This data is not a sample but a population of eight. The mean is as follows: μ = (9.0 + 9.5 + 9.3 + 7.2 + 10.0 + 9.1 + 9.4 + 9.0) / 8 = 72.5 / 8 = 9.063 ◆ Since the number of data is N = 8 which is an even number, the median is the average of the 4^{th} and the 5^{th} data in the ordered list as follows: 7.2 9.0 9.0 9.1 9.3 9.4 9.5 10.0 Therefore, the median is the average of 9.1 and 9.3 which is 9.2. ◆ The trimmed mean is the average of the remaining numbers, except the minimum of 7.2 and the maximum of 10.0. Trimmed Mean = (9.0 + 9.0 + 9.1 + 9.3 + 9.4 + 9.5) / 6 = 55.3 / 6 = 9.217 ◆ In this data, the median or the trimmed mean is more representative of the data than the arithmetic mean.

Example 4.3.4	**(Weighted Mean)** A student took three courses in History (two credits), Math (four credits), and English (three credits) in last semester, and got A in History, B in math and C in English. Calculate the mean and the weighted mean with the number of credits if A is rated 4 points, B is 3 points, and C is 2 points.
Answer	◆ Mean = (4 + 3 + 2) / 3 = 3 ◆ Weighted Mean = $\frac{2\times4+4\times3+3\times2}{2+4+3}=\frac{8+12+6}{9}=2.89$ ◆ Weighted mean is less than mean, because although the grade of History which has two credits was A, the grade of English which has three credits was relatively poor C.

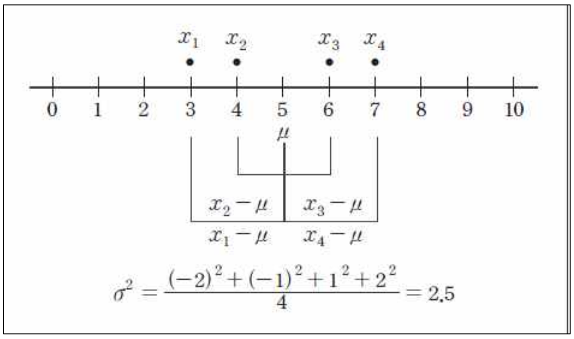
4.3.2 Measures of Dispersion

- In a gymnastics competition, four judges scored 3, 4, 6, and 7 points for a player A and 2, 4, 6, and 8 points for a player B. Both players have the same mean of 5, but it is easy to see that the player B has a large deviation in the scores compared to the player A. Degree of data dispersion is calculated using a numerical value to compare two sets of data and it is called a measure of dispersion. The most commonly used measure of dispersion is a variance (or standard deviation) and other measures include a mean absolute deviation, a range, and an inter-quartile range.
- A **variance** is an average of all squared distances from each data to the mean. Therefore, if data are spread widely around their mean, the variance will be large, and if data are concentrated around the mean, the variance will be small. A population variance is denoted as σ^2, and a sample variance is denoted as s^2. Formulas to calculate the population variance and the sample variance are slightly different as follows:

Population variance $\sigma^2 = \dfrac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}$ (N: number of population data)

Sample variance $s^2 = \dfrac{\sum_{i=1}^{n}(x_i-\overline{x})^2}{n-1}$ (n: number of sample data)

- There are important reasons for using $n-1$ instead of n when calculating the sample variance which will be discussed in Chapter 6. Meaning of the population variance, which is an average of all squared distances from each data to the population mean, is illustrated in <Figure 4.3.6>. In this Figure, ● mark represents each data value. σ^2 = 2.5 is calculated as the sum of squared distances (10) divided by the number of data, n=4 in this example.



<Figure 4.3.6> Calculation of a population variance

- A **standard deviation** is defined as the square root of the variance. A population standard deviation is denoted as σ, and a sample standard deviation is denoted as s. The variance is not easy to interpret, because it is an average of the squared distances. However, since the standard deviation is the square root of the variance, it is interpreted as an average distance from each data value to the mean.

Population standard deviation $\sigma = \sqrt{\sigma^2}$

Sample standard deviation $s = \sqrt{s^2}$

Definition	**Variance and Standard Deviation** A **variance** is an average of all squared distances from each data to the mean. A population variance is denoted as σ^2, and a sample variance is denoted as s^2. A **standard deviation** is defined as the square root of the variance. A population standard deviation is denoted as σ, and a sample standard deviation is denoted as s.

Example 4.3.5	In Example 4.3.1, the mean of the following sample data was calculated as 6. 5, 6, 3, 7, 9, 4 and 8. Calculate a sample variance and a sample standard deviation of this data.
Answer	◆ The sample mean was calculated as follows: $\overline{x}$ = (5 + 6 + 3 + 7 + 9 + 4 + 8) / 7 = 6. ◆ Since this data are sampled, the sample variance is calculated as follows. Note that it is divided by (7-1). $s^2 = \frac{(5-6)^2+(6-6)^2+(3-6)^2+(7-6)^2+(9-6)^2+(4-6)^2+(8-6)^2}{(7-1)} = \frac{28}{6} = 4.667$ ◆ The sample standard deviation is the square root of the sample variance s^2. $s = \sqrt{s^2} = \sqrt{4.667} = 2.16$ ◆ These values coincide with the output of 『eStat』 in <Figure 4.3.3> and the output of 『eStatU』 in <Figure 4.3.4>.

- When there are more than two quantitative variables, if units of data measurement are different from each other, comparing their standard deviations is meaningless. In this case, a **coefficient of variation** which is a division of the standard deviation by the mean, is used to compare several variables. The coefficient of variation is usually calculated as a percent value of the standard deviation to its mean.

Population Coefficient of Variation	$C = \frac{\sigma}{\mu} \times 100$	(unit %)
Sample Coefficient of Variation	$c = \frac{s}{\overline{x}} \times 100$	(unit %)

Definition	**Coefficient of variation** A **coefficient of variation** is a division of the standard deviation by the mean and it is used to compare several variables. The coefficient of variation is usually calculated as a percent value of the standard deviation to its mean.

Example 4.3.6	**(Sales data)** In a company, the average weekly sales was 1.36 billion dollar and its standard deviation was 0.28 billion dollar. If the same data were made in monthly sales, the average was 5.44 billion dollar and its standard deviation was 0.5 billion dollar. Calculate a coefficient of variation for each case and compare.
Answer	◆ The coefficient of variation in weekly sales is as follows: (0.28 / 1.36) × 100 = 20.6%, ◆ The coefficient of variation in monthly sales is as follows: (0.50 / 5.44) × 100 = 9.2%. ◆ Therefore, we can see that the variation in monthly sales is smaller than the variation in weekly sales.

- A **range** is the difference between the maximum and the minimum value of data. The range is easy to calculate, but it is not a good measure of dispersion if there are extreme points.

  Range = Maximum - Minimum

- A **p-percentile** implies roughly the p^{th} percent data when data are arranged in ascending order from small to large.

  p percentile = there are p% of observations less than or equl to (≦) this value
  and (100-p)% of observations located above or equal to (≧) this value .

  Note that, if data size is small, a single observation may fall into several percentiles according to this definition.
- An **inter-quartile range** is a measure to complement the disadvantage of the range. The 25 percentile of the data is called the 1^{st} quartile (Q1), the 50 percentile is called the 2^{nd} quartile (Q2) or median, and the 75 percentile is called the 3^{rd} quartile (Q3). The inter-quartile range (IQR) is the range between the 3^{rd} quartile and the 1^{st} quartile.

  Inter-quartile range (IQR) = Q3 - Q1

  One simple way to calculate Q1 and Q3 is that, after we arrange the data in ascending order, we divide the data into two pieces which have equal number of data. In case of odd number of data, we include the median to each piece of data. Q1 is the median of the 1^{st} piece of data and Q3 is the median of the 2^{nd} piece of data.

Definition

Range, percentile, Quartile and Inter-quartile Range

A **range** is the difference between the maximum and the minimum value of data.

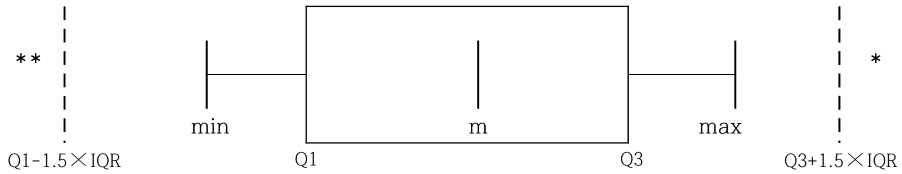
A **p-percentile** is that there are p% of data less than or equaa to (≦) this value and (100-p)% of data located above or equal to (≧) this value. The 25 percentile of the data is called the **1st quartile** (Q1), the 50 percentile is called the 2nd quartile (Q2) or median, and the 75 percentile is called the **3rd quartile** (Q3).

An **inter-quartile range** (IQR) is the range between the 3rd quartile and the 1st quartile.

Example 4.3.7	If you have data 5, 3, 7, 9, find a range and an inter-quartile range.
Answer	◆ The maximum of the data is 9 and the minimum is 3, therefore, range is as follows: Range = 9 – 3 = 6. ◆ In order to find the quartiles of the data, first arrange the data in ascending order as follows: 3, 5, 7, 9. ◆ The median of these numbers is the average of $(\frac{4}{2})^{th}$ and $(\frac{4}{2}+1)^{th}$. Median = (5 + 7) / 2 = 6. ◆ In order to calculate quartiles, since the number of data is even, we divide data into two pieces as follows: {3, 5} {7, 9} ◆ The first quartile Q1 is the median of {3, 5}. Q1 = 4 The third quartile Q3 is the median of {7. 9}. Q3 = 8. So, the inter-quartile range IQR is as follows: IQR = Q3 - Q1 = 8 - 4 = 4.

Example 4.3.8	Using the data of Example 4.3.1 which are as follows, calculate a range and an inter-quartile range and compare it with the output of 『eStat』. 5, 6, 3, 7, 9, 4 and 8.
Answer	◆ The maximum of the data is 9 and the minimum is 3. Therefore, the range is as follows: Range = 9 – 3 = 6. ◆ In order to find quartiles of data, first arrange the data in ascending order as follows: 3, 4, 5, 6, 7, 8, 9. ◆ The median of the data is the data value of $(\frac{7+1}{2})^{th} = 4^{th}$ which is 6. ◆ In order to calculate the quartiles, since the number of data is odd, divide the data into two pieces as follows. Note that the median is included in both pieces of data. {3, 4, 5, 6} {6, 7, 8, 9} ◆ The first quartile Q1 is the median of {3, 4, 5, 6} which is Q1 = 4.5 The third quartile Q3 is the median of [6, 7. 8, 9] which is Q3 = 7.5. So, the inter-quartile range IQR is as follows: IQR = Q3 - Q1 = 7.5 – 4.5 = 3. ◆ These values of Q1, Q3 and IQR coincide with the output of 『eStat』 in <Figure 4.3.3> and the output of 『eStatU』 in <Figure 4.3.4>.

- A **box plot** is a graph to show the minimum, the 1st quartile, the median, the 3rd quartile, and the maximum of the data simultaneously that has been used recently. The box plot first marks the 1st quartile (Q1) and the 3rd quartile (Q3) at a horizontal line and connects with a square box. Then displays the median (Q2) at the location proportional to Q1 and Q3 in the box and connects the box with the minimum and the maximum. Also, draw a vertical line at (minimum - 1.5×IQR) and at (maximum + 1.5×IQR) as in <Figure 4.3.3>. Using the box plot, you can check a symmetry of data, a central location of data (median), and a degree of dispersion (IQR). Data which are less than the line (minimum - 1.5× IQR) or greater than (maximum + 1.5×IQR) are considered as extremes (marked * in <Figure 4.3.7>). Some statistical packages display the left line which is to check an extreme point as Max(minimum, Q1 - 1.5×IQR) and the right line as Min(maximum, Q3 + 1.5×IQR).



<Figure 4.3.7> Box plot

Definition	**Box Plot** A **box plot** is a graph to show minimmum, Q1, median, Q3, maximum of data simultaneously that has recently begun to be widely used.

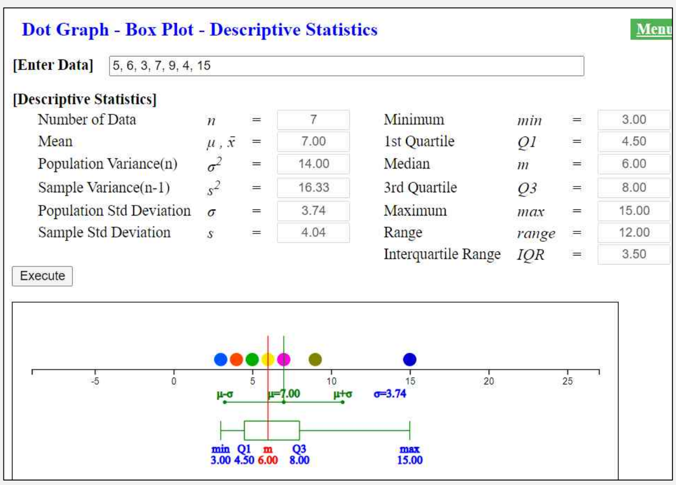
Example 4.3.9

Using the following data, draw a dot plot and a box plot using『eStatU』.

5, 6, 3, 7, 9, 4, 15.

Answer

- Using the menu 'Dot Graph – Box Plot – Descriptive Statistics' in『eStatU』, if you enter the data and click the [Execute] button, the dot plot and the box plot appear as in <Figure 4.3.8>.



<Figure 4.3.8> Dot graph and box plot of the data

Example 4.3.10

(Ages of teachers by gender)

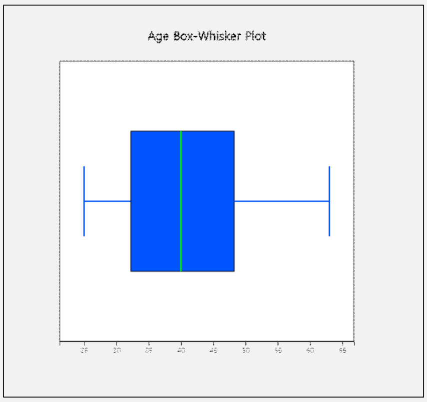
In a middle school, ages of all teachers with their gender were surveyed and the data can be found at the following location of 『eStat』.

Ex ⇨ eBook ⇨ EX040310Continous_TeacherAgeByGender.csv

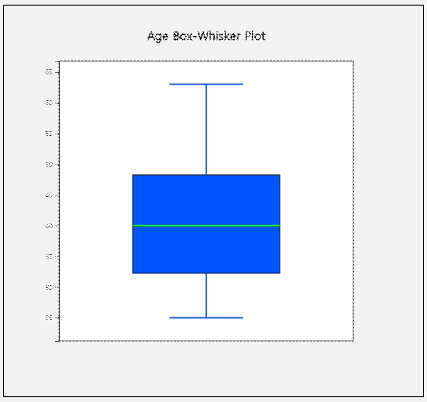
1) Draw a box plot of the age using 『eStat』 and examine a median, a range, a quartile and an inter-quartile range.
2) Draw a box plot of the age by gender using 『eStat』 and compare medians, ranges, quartiles and IQRs by gender.

Answer

1) After loading the data to 『eStat』, enter the value labels of 'Gender' as 'Male' for 1 and 'Female' for 2 using [EditVar] button. Click on the box plot icon and then 'Age' variable, then the horizontal box plot is appeared as in <Figure 4.3.9>. If you select 'Vertical' from the options below the graph, the vertical box plot shown as in <Figure 4.3.10> is appeared. Based on this box plot, we can see that the upper half of data is more scattered than the lower half of data which implies there are more aged teachers.



<Figure 4.3.9> Horizontal box plot of age variable



<Figure 4.3.10> Vertical box plot of age variable

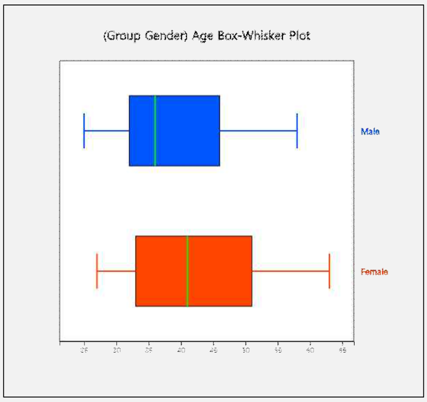
- If you click button of [Descriptive Statistics] in the options, the basic statistics of the age is displayed as shown in <Figure 4.3.11>.

Descriptive Statistics	Analysis Var (Age)
Observation	30
Missing Observations	0
Mean	40.667
Variance (n)	116.822
Variance (n-1)	120.851
Std Dev (n)	10.808
Std Dev (n-1)	10.993
Minimum	25.000
1st Quartile	32.250
Median	40.000
3rd Quartile	48.250
Maximum	63.000
Range	38.000
Interquartile Range	16.000
Coefficient of Variation (n)	26.58 %
Coefficient of Variation (n-1)	27.03 %

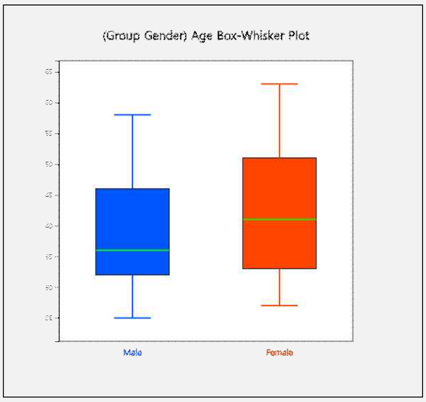
<Figure 4.3.11> Descriptive Statistics of age

Example 4.3.10 Answer (continued)

2) If you click on 'Gender' after 'Age' variable, the horizontal box plot by gender appears as shown in <Figure 4.3.12>. If you select 'Vertical' from the options below the graph, the vertical box plot by gender appears as shown in <Figure 4.3.13>. You can observe that dispersion of female teachers' ages is greater than that of male teachers'.



<Figure 4.3.12> Horizontal box plot of age by gender



<Figure 4.3.13> Vertical box plot of age by gender

- If you click the button of [Basic Statistics] in the options, the basic statistics of the age by gender is displayed in the Log Area as in <Figure 4.3.14>.

Descriptive Statistics	Analysis Var (Age)	Group Name (Gender) 1 (Group 1)	Group Name (Gender) 2 (Group 2)
Observation	30	13	17
Missing Observations	0		
Mean	40.667	38.846	42.059
Variance (n)	116.822	106.592	120.173
Variance (n-1)	120.851	115.474	127.684
Std Dev (n)	10.808	10.324	10.962
Std Dev (n-1)	10.993	10.746	11.300
Minimum	25.000	25.000	27.000
1st Quartile	32.250	32.000	33.000
Median	40.000	36.000	41.000
3rd Quartile	48.250	46.000	51.000
Maximum	63.000	58.000	63.000
Range	38.000	33.000	36.000
Interquartile Range	16.000	14.000	18.000
Coefficient of Variation (n)	26.58 %	26.58 %	26.06 %
Coefficient of Variation (n-1)	27.03 %	27.66 %	26.87 %

<Figure 4.3.14> Descriptive statistics of age by gender

[Practice 4.3.2]

(Effect of Vitamin C on Tooth Growth in Guinea Pigs)

The effect of Vitamin C on tooth growth in Guinea Pigs was examined. The response is the length of odontoblasts (cells responsible for tooth growth) in 60 guinea pigs. Each animal received one of three dose levels of vitamin C (0.5, 1, and 2 mg/day) by one of two delivery methods, orange juice or ascorbic acid (a form of vitamin C and coded as VC). Data can be found at the following location of 『eStat』.

Ex ⇨ eBook ⇨ PR040302_Rdatasets_ToothGrowth.csv

Data format:

V1	lenth	numeric	Tooth length
V2	supp	factor	Supplement type (VC or OJ).
V3	dose	numeric	Dose in milligrams/day

1) Draw a box plot of the length using 『eStat』 and find the median, the range, the quartiles and the IQR. Analyze the graph and the basic statistics.
2) Draw a box plot of the length by the supplement using 『eStat』 and find the median, the range, the quartiles and the IQR by the supplement. Analyze the graphs and the basic statistics.
3) Draw a box plot of the length by the dose using 『eStat』 and find the median, the range, the quartiles and the IQR by the doset. Analyze the graphs and the basic statistics.

Exercise

4.1 Mid term scores of a Statistics course are 70, 60, 80, 90, 90, 70. What is the mean and the median of this data?

4.2 There are cards that write numbers 1, 2, 3, ... , n. What is the average of these numbers?

4.3 For data measured as 2, 3, 7, 7, 7, 7, 8, find the mean, the median and the mode.

4.4 The following table is the evaluation scores of the courses taken by a student this semester. What is the weighted mean of these scores by using the credits as their weights?

Course Name	A	B	C	D
Credit	4	3	2	1
Score	80	90	75	90

4.5 Test scores of 10 students which we sampled from all students of Statistics course were 6, 8, 7, 8, 5, 9, 7, 10, 9, 4. What is the sample mean and the sample standard deviation?

4.6 Life expectancies of 10 different automobiles sampled from a population were investigated as follows: (unit year)

3 3 8 7 4 6 5 2 5 10

Calculate
1) mean, 2) median, 3) mode, 4) variance and standard deviation,
5) coefficient of variation, 6) range, 7) inter-quartile range.

4.7 After sampling 10 employees from a company, we examined commuting distances (km) from their home to the company and found the following data.

3 16 12 11 14 5 7 14 9 8

Calculate
1) mean, 2) median, 3) mode, 4) variance and standard deviation,
5) coefficient of variation, 6) range, 7) inter-quartile range.

4.8 The following is a list of stock prices of a company during the last 25 days of closing. (Unit: US$)

131, 135, 129, 123, 130, 136, 134, 140, 146, 150,
153, 150, 148, 151, 153, 158, 161, 165, 160, 155,
157, 163, 159, 160, 160.

Use『eStat』to do the followings.

1) Calculate the mean, the median and the mode for the above data.
2) Obtain the weighted average by weighting 25 on the stock price of the most recent work, then 24 on the next stock price, ... and 1 on the stock price of the

first date. Compare the mean value obtained in 1) with that value.
3) Calculate the variance and the standard deviation, the coefficient of variation, the range, the inter-quartile range.
4) Calculate the 1st quartile (Q1) and the 3rd quartile (Q3).
5) Draw a box plot.

4.9 Scores of two bowling players playing 10 games were as follows:

Player A	Player B
198	196
119	159
174	162
235	178
134	188
192	169
124	173
241	183
158	177
176	152

Use『eStat』to do the followings.

1) Calculate the mean and the median for each player.
2) Find the standard deviation, the range, the 1st quartile, the 3rd quartile and the inter-quartile range for each player.
3) Draw a box plot.
4) Who do you think is the better player? Why?

4.10 To test the effectiveness of a memory improvement technique developed by a psychologist, 30 samples observed the difference in time taken to memorize 10 numerical sequences of 10 pairs before and after learning the technique, as shown below. (Unit: Minutes)

5, 10, 15, 11, 13, 20, 14, 5, 23, 18, 17, 4, 19, 5, 24, 18, 15, 21,
24, 16, 2, 15, 19, 22, 24, 21, 14, 18, 26, 10.

Use『eStat』to do the followings.

1) Draw a histogram of the above data. Find the frequency table of the histogram.
2) Calculate the mean and the median and compare their values.
3) Calculate the quartiles and draw a box plot.

Multiple Choice Exercise

4.1 Which of the following data is an average of 28, a median of 30, and a maximum of 40?

① 12, 20, 30, 40 ② 12, 30, 30, 40
③ 12, 40, 30, 40 ④ 12, 40, 20, 40

4.2 Six statistical scores are 70, 60, 80, 90, 90, 70. What is the median value of these scores?

① 70 ② 75 ③ 80 ④ 90

4.3 Numbers 1, 2, 3, ...There are cards that write each and every one of them. What is the average of these numbers?

① $\frac{(n+1)(2n+1)}{2}$ ② $\frac{(2n+1)}{3}$
③ $\frac{n(n+1)}{2}$ ④ $\frac{(n+1)}{2}$

4.4 I bought 10 tomatoes which cost 1 dollar each and 10 tomatoes which cost 2 dollars each. How much is its cost in average for each?

① 1.5 ② 1 ③ 2 ④ 1.3

4.5 If the averages of two data sets are $\overline{x}_1$, $\overline{x}_2$ and their data sizes are n_1, n_2, what is the average of the total data combined?

① $\frac{n_1 \overline{x}_1 + n_2 \overline{x}_2}{n_1 + n_2}$ ② $\frac{n_1 \overline{x}_2 + n_2 \overline{x}_1}{n_1 + n_2}$
③ $\frac{\overline{x}_1 + \overline{x}_2}{n_1 + n_2}$ ④ $\frac{n_1 n_2 (\overline{x}_1 + \overline{x}_2)}{n_1 + n_2}$

4.6 If data are X_1, X_2, X_3, $\cdots$, X_n and its mean is $\overline{X}$, what is the value of $\sum_{j=1}^{n}(X_j - \overline{X})$?

① 1 ② 0 ③ -1 ④ n

4.7 Which of the following properties of the mean is incorrect?

① The mean is greatly influenced by the extreme value of the data.
② The sum of the deviations from the mean is not zero.
③ The sum of the deviations from the mean is zero.
④ The mean is a measure of the central tendency.

4.8 The following table is the evaluation scores of a university student. What is the weighted average of the scores using the credits as weights?

Course	A	B	C	D	E	F
Credit	4	3	3	2	2	1
Score	80	90	85	95	75	90

① 82.85 ② 85.00 ③ 83.25 ④ 80.00

4.9 Which of the following statistical analysis is wrong if the 1^{st} quartile is 68.25 and the 2^{nd} quartile is 79.06 and the 3^{rd} quartile is 90.75?

① 25% of the total number is 68.25 or less.
② 50% of the total frequency is 68.25 or less.
③ 50% of the total frequency is 79.06 or less.
④ 75% of the total frequency is 90.75 or below.

4.10 What is a convenient measure to compare the dispersion of data which has different units?

① relative frequency
② standard deviation
③ coefficient of variation
④ correlation coefficient

(Answers)
4.1 ④, 4.2 ②, 4.3 ④, 4.4 ①, 4.5 ①, 4.6 ②, 4.7 ②, 4.8 ②, 4.9 ②, 4.10 ③